



Finite Field-Dependent BRST Transformations as Quantum Canonical Flows in the Batalin-Vilkovisky Formalism

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Abstract. We reformulate finite field-dependent BRST transformations as quantum canonical flows in the Batalin-Vilkovisky field-antifield formalism. The starting point is not an ansatz for the Jacobian but a path between two gauge-fixing fermions. The change of gauge is represented by a one-parameter family of BV Lagrangian submanifolds, while a field-dependent BRST transformation is identified with the field-space projection of a canonical flow generated by the gauge-fixed BV master action. This leads to a gauge-fermion transgression equation in which the Jacobian and the variation of the gauge-fixing fermion are two components of one quantum canonical transport law.

For an anomaly-free theory satisfying the quantum master equation, we show that the partition function is independent of the chosen gauge-fixing path. We then apply the construction to non-Abelian Yang-Mills theory and to the reducible Abelian rank-two antisymmetric tensor theory. In the latter case, the complete ghost and ghost-of-ghost hierarchy is incorporated naturally in the BV master action. Covariant/Lorentz and axial gauges appear as two endpoints of a single BV path. We derive a compact compensation condition for finite field-dependent transformations and identify the precise role of the BV Laplacian in the Jacobian. The resulting framework clarifies the relation between FFBRST transformations, gauge-fixing canonical transformations, and the quantum master equation. It also shows why a local Jacobian cannot be assumed in general: locality is a property of the chosen generator and regularization, not of finite field dependence by itself.

Keywords: finite field-dependent BRST transformations; Batalin-Vilkovisky formalism; gauge fixing; quantum canonical transformations; BRST-BV symmetry; Jacobian.

Received 4.09.2026. Revised 9.09.2026. Accepted 9.09.2026. Available online 30.09.2026

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Introduction

Gauge theories provide the fundamental framework for describing electromagnetic, weak, and strong interactions, with the non-Abelian gauge structure of the Standard Model formulated within Yang-Mills (YM) theory [1]. A characteristic feature of gauge theories is the redundancy in their description of physical configurations, which necessitates an appropriate treatment of gauge symmetry during quantization. Gauge fixing is therefore an essential step in constructing a well-defined quantum theory. Among the various possible choices, Lorentz-type and axial-type gauges are particularly important. Lorentz-type gauges preserve manifest covariance and provide a convenient framework for perturbative calculations and the construction of Feynman rules [2]; however, their quantization generally involves a nontrivial Faddeev-Popov ghost sector, which may increase the complexity of practical calculations [3]. Axial-type gauges provide an alternative formulation in which the ghost contribution can become trivial under suitable conditions, thereby simplifying the unphysical sector [4]. Understanding the relation between different gauge-fixed formulations is consequently an important problem in gauge theory. The issue becomes more subtle in reducible gauge systems, such as the Abelian antisymmetric rank-two tensor theory [5,6], where the gauge transformations themselves possess redundancies. A consistent quantization of such systems requires additional ghost fields, including ghosts of ghosts. Antisymmetric tensor and higher-form gauge fields are also of considerable physical interest because of their appearance in string theory [6], fluid dynamics [7,8], dual formulations of Higgs models [9,10], supergravity [11], and superstring theories [12,13].

The Batalin-Vilkovisky (BV) field-antifield formalism provides a systematic and unified framework for the quantization of gauge theories with both irreducible and reducible gauge symmetries [14-17]. In the BV formulation, gauge fields, ghosts, ghosts of ghosts, auxiliary fields, and their corresponding antifields are incorporated into an extended field-antifield space, while the complete gauge structure of the theory is encoded in the BV master action. The consistency of the gauge symmetry is governed by the master equation, while its quantum extension incorporates the contribution of the functional measure through the quantum master equation. One of the essential features of the BV formalism is that gauge fixing is formulated through the choice of a gauge-fixing fermion, which specifies a Lagrangian submanifold in the extended field-antifield space. Consequently, different gauge choices can be viewed as different gauge-fixing realizations of the same underlying BV theory rather than as distinct theories. This provides a particularly natural description of reducible systems, since the complete hierarchy of ghosts and higher-generation ghosts is incorporated into the same master-action framework. The BRST symmetry, discovered independently by Becchi, Rouet and Stora [18] and Tyutin [19], is naturally embedded in the BV construction and provides an important tool for the analysis of renormalization, unitarity, and the physical sector of gauge theories [14,15,18,19]. Thus, the BV formalism establishes a direct connection between gauge symmetry, gauge fixing, BRST symmetry, reducibility, and the quantum measure.

The finite field-dependent BRST (FFBRST) transformation, originally introduced by Joglekar and Mandal [20], provides a powerful extension of the conventional BRST framework. In an FFBRST transformation, the BRST parameter is promoted from an infinitesimal constant quantity to a finite field-dependent functional. Such a transformation generally modifies the functional measure and produces a nontrivial Jacobian in the path integral. Under appropriate conditions, this Jacobian can be represented by an additional contribution to the effective action and can establish a connection between different gauge-fixed formulations. The FFBRST method has subsequently been applied to a wide variety of gauge theories and related systems [21-33], while alternative formulations of field-dependent BRST transformations have also been developed [34,35]. Finite BRST-anti-BRST transformations provide a further generalization and have been extensively studied in Yang-Mills and more general gauge theories [36-39]. In the conventional FFBRST approach, the finite transformation is usually constructed first, and its Jacobian is subsequently evaluated in order to determine the corresponding change in the effective action. The BV formalism suggests a complementary and more geometric organization of this procedure. Since a gauge choice is characterized by a gauge-fixing fermion, a finite transformation between two gauges can be regarded as a deformation between the corresponding gauge-fixing structures in the field-antifield space. A continuous family of gauge-fixing fermions therefore defines a path through the space of gauge choices, and the associated deformation of the BV Lagrangian submanifold can be interpreted as a canonical flow. Its projection onto the original field space naturally gives a field-dependent BRST transformation, while the associated change of the functional measure is governed by the

quantum structure of the BV formalism.

In the present work, we develop this BV-based interpretation of finite field-dependent BRST transformations by taking the gauge-fixing fermion as the primary object defining the transformation between two gauge choices. Rather than treating the functional Jacobian as an independent quantity to be reconstructed after specifying the transformation, we formulate the finite field-dependent transformation as the field-space projection of a quantum canonical flow connecting two BV gauge-fixing structures. This provides a unified description of gauge fixing, BRST symmetry, finite field dependence, and quantum measure effects within the field-antifield formalism. The construction is first applied to Yang-Mills theory to establish the relation between Lorentz-type and axial-type gauges. It is then extended to the Abelian antisymmetric rank-two tensor theory, where the reducible gauge structure provides a nontrivial test of the formulation and where the ghost and ghost-of-ghost sectors are incorporated simultaneously through the BV master action. In this framework, the change of gauge is understood as a continuous deformation in gauge-fixing space, while the corresponding FFBRST transformation represents its projection onto the original field space. The quantum master equation provides the consistency condition for the associated measure transformation, and the locality of a compensating Jacobian is treated as a property that must be established for the chosen transformation and regularization rather than assumed a priori. This formulation therefore provides a geometrical perspective on FFBRST transformations as finite gauge-fixing flows in the BV field-antifield space and offers a systematic framework for studying finite changes of gauge in both irreducible and reducible gauge theories.

BV geometry and gauge-fixing paths

Fields, antifields and the antibracket

Let the complete set of fields be denoted by

$$\phi^A, \quad (1)$$

and introduce an antifield ϕ_A^* for every field. The Grassmann parity satisfies

$$\varepsilon(\phi_A^*) = \varepsilon(\phi^A) + 1 \pmod{2} \quad (2)$$

The BV antibracket is

$$(F, G) = \frac{\overleftarrow{\delta} F}{\delta \phi^A} \frac{\overrightarrow{\delta} G}{\delta \phi_A^*} - \frac{\overleftarrow{\delta} F}{\delta \phi_A^*} \frac{\overrightarrow{\delta} G}{\delta \phi^A}. \quad (3)$$

It is an odd Poisson bracket and obeys graded antisymmetry, the graded Jacobi identity and the graded Leibniz rule.

The classical BV master action S obeys

$$(S, S) = 0. \quad (4)$$

At the quantum level, with $\hbar$ restored, the master action W satisfies the quantum master equation

$$\frac{1}{2} (W, W) - i\hbar \Delta W = 0, \quad (5)$$

where the BV Laplacian is formally

$$\Delta = (-1)^{\varepsilon_A} \frac{\overleftarrow{\delta}}{\delta \phi^A} \frac{\overrightarrow{\delta}}{\delta \phi_A^*}. \quad (6)$$

The precise definition of Δ requires a regularization prescription. This point will be important below.

Gauge fixing as a Lagrangian submanifold

A gauge-fixing fermion $\Psi[\phi]$ has ghost number -1 and odd Grassmann parity. Gauge fixing is implemented through

$$\phi_A^* = \frac{\bar{\delta}\Psi}{\delta\phi^A}. \quad (7)$$

The gauge-fixed quantum action is the restriction

$$W_\Psi[\phi] = W\left[\phi, \phi^* = \frac{\delta\Psi}{\delta\phi}\right]. \quad (8)$$

For a classical solution of the master equation, the induced BRST differential is

$$s_\Psi F = (F, W_\Psi) \quad (9)$$

in the standard reduced notation.

The key geometric fact is that changing Ψ changes the Lagrangian submanifold rather than the underlying BV theory. Let

$$\Psi_\kappa = \Psi_0 + \int_0^\kappa d\kappa' \dot{\Psi}_{\kappa'}. \quad (10)$$

Then

$$\dot{W}_\kappa = \frac{dW_{\Psi_\kappa}}{d\kappa} \quad (11)$$

is a gauge-fixing deformation. In an anomaly-free theory, its contribution to the vacuum functional is BRST/BV exact.

Gauge-space path

For two gauges labelled by 0 and 1 define

$$\Psi_\kappa = (1 - \kappa)\Psi_0 + \kappa\Psi_1. \quad (12)$$

Then

$$\dot{\Psi}_\kappa = \Psi_1 - \Psi_0. \quad (13)$$

The present proposal is to regard (12) as the primary input. The finite field-dependent transformation is constructed so that its endpoint is the field-space projection of this BV transport.

Quantum canonical FFBRST flow*From BRST flow to BV flow*

The ordinary FFBRST flow is

$$\frac{d\phi^A(\kappa)}{d\kappa} = s_\kappa\phi^A(\kappa) \Theta_\kappa[\phi(\kappa)], \quad (14)$$

where Θ_κ is Grassmann odd. In the BV space, introduce a generator

$$G_\kappa = W_\kappa \Theta_\kappa. \quad (15)$$

When Θ_κ depends only on fields and the chosen sign convention is used, the field projection of the anticanonical flow generated by G_κ contains

$$\frac{d\phi^A}{d\kappa} = (\phi^A, G_\kappa) = (\phi^A, W_\kappa)\Theta_\kappa. \quad (16)$$

Thus the ordinary field-dependent BRST flow appears as a projection of a canonical BV flow.

Equation (16) is not by itself the complete quantum transformation. A field-dependent canonical transformation changes the measure as well. The BV Laplacian supplies the quantum correction required to describe this change consistently.

Quantum canonical transformation

For an odd generator G_κ , define the infinitesimal quantum canonical action on a functional F by

$$\frac{dF}{d\kappa} = (F, G_\kappa) - i\hbar \Delta G_\kappa. \quad (17)$$

The second term is the measure contribution. In a regulated formulation, it is understood as the corresponding regulated BV divergence.

For the path integral

$$Z_\kappa = \int_{\mathcal{L}_\kappa} D\phi \exp\left(\frac{i}{\hbar} W\right), \quad (18)$$

the combined variation of the integration Lagrangian submanifold and the measure is controlled by the quantum canonical flow. The fundamental compensation relation can be written schematically as

$$\frac{dW_{\Psi_\kappa}}{d\kappa} = (W_{\Psi_\kappa}, G_\kappa) - i\hbar \Delta G_\kappa. \quad (19)$$

The precise sign of the Δ term follows the convention for the BV measure and the quantum canonical transformation.

For a pure gauge-fixing deformation generated by Ψ_κ , the corresponding quantum canonical variation reduces to the standard BV gauge-independence identity. In an anomaly-free regularization,

$$\frac{dZ_\kappa}{d\kappa} = 0. \quad (20)$$

BV transgression formula

The central result can be expressed in endpoint form. Let $\mathcal{J}_\kappa$ denote the Jacobian associated with the projected field-dependent transformation. Then

$$\frac{d}{d\kappa} \ln \mathcal{J}_\kappa = \frac{i}{\hbar} \frac{d}{d\kappa} (W_{\Psi_\kappa} - W_{\Psi_0}), \quad (21)$$

provided the quantum master equation is satisfied, and the same measure prescription is used along the path. Hence

$$\ln \mathcal{J}_1 = \frac{i}{\hbar} (W_{\Psi_1} - W_{\Psi_0}). \quad (22)$$

Equivalently,

$$\mathcal{J}_1 = \exp\left[\frac{i}{\hbar} (W_{\Psi_1} - W_{\Psi_0})\right]. \quad (23)$$

This equation should be understood as a compensation identity, not as a universal statement that every field-dependent transformation has a local Jacobian. The right-hand side is local only when the chosen gauge-fixing deformation and the regulated BV divergence admit a local representation.

Why the gauge-fixing fermion is the natural variable

In the Faddeev-Popov formulation, two gauges appear as two different effective actions. In BV they are two restrictions of the same master action:

$$W_{\Psi_0} = W|_{\mathcal{L}_{\Psi_0}}, \quad W_{\Psi_1} = W|_{\mathcal{L}_{\Psi_1}}. \quad (24)$$

Therefore, the problem of connecting gauges is geometrically the problem of transporting one Lagrangian submanifold into another. FFBRST becomes the field-space shadow of this transport.

This is the principal conceptual reorganization of the method.

Yang-Mills theory

Minimal BV master action

For Yang-Mills fields A_μ^α and ghosts c^α , the minimal BV master action is

$$S_{min} = S_{YM} + \int d^4x \left[A_\alpha^{*\mu} D_\mu^{\alpha\beta} c^\beta - \frac{g}{2} c_\alpha^* f^{\alpha\beta\gamma} c^\beta c^\gamma \right], \quad (25)$$

where

$$S_{YM} = -\frac{1}{4} \int d^4x F_{\mu\nu}^\alpha F^{\alpha\mu\nu}. \quad (26)$$

The master equation

$$(S_{min}, S_{min}) = 0 \quad (27)$$

flows from the closure of the Yang-Mills gauge algebra and the Jacobi identity of the structure constants.

For a practical gauge-fixed path integral one introduces the non-minimal pair $(\bar{c}^\alpha, B^\alpha)$ and their antifields. A convenient non-minimal contribution is

$$S_{nm} = \int d^4x \bar{c}_\alpha^* B^\alpha. \quad (28)$$

The complete proper solution is

$$S = S_{min} + S_{nm}. \quad (29)$$

Lorentz gauge

Choose the gauge-fixing fermion

$$\Psi_L = \int d^4x \bar{c}^\alpha \left(\partial^\mu A_\mu^\alpha + \frac{\lambda}{2} B^\alpha \right). \quad (30)$$

The antifield elimination gives

$$A_\alpha^{*\mu} = \partial^\mu \bar{c}_\alpha, \quad (31)$$

$$\bar{c}_\alpha^* = \partial^\mu A_\mu^\alpha + \frac{\lambda}{2} B^\alpha, \quad (32)$$

$$c_\alpha^* = 0, \quad (33)$$

$$B_\alpha^* = \frac{\lambda}{2} \bar{c}_\alpha. \quad (34)$$

The resulting gauge-fixed action is equivalent, up to convention-dependent signs, to η^μ

$$S_L = S_{YM} + \int d^4x \left[B^\alpha \partial^\mu A_\mu^\alpha + \frac{\lambda}{2} (B^\alpha)^2 - \bar{c}^\alpha \partial^\mu D_\mu^{\alpha\beta} c^\beta \right]. \quad (35)$$

Axial gauge and BV interpolation

Let η^μ be a fixed constant vector and choose

$$\Psi_A = \int d^4x \bar{c}^\alpha \left(\eta^\mu A_\mu^\alpha + \frac{\lambda}{2} B^\alpha \right). \quad (36)$$

The interpolation is

$$\Psi_\kappa = (1 - \kappa)\Psi_L + \kappa\Psi_A, \quad (37)$$

or

$$\Psi_\kappa = \int d^4x \bar{c}^\alpha \left[G_\kappa^\alpha + \frac{\lambda}{2} B^\alpha \right], \quad (38)$$

with

$$G_\kappa^\alpha = (1 - \kappa) \partial^\mu A_\mu^\alpha + \kappa \eta^\mu A_\mu^\alpha. \quad (39)$$

The tangent vector in gauge space is

$$\dot{\Psi}_\kappa = \int d^4x \bar{c}^\alpha (\eta^\mu A_\mu^\alpha - \partial^\mu A_\mu^\alpha). \quad (40)$$

FFBRST parameter from the BV path

The field-space projection of the BV transport suggests the finite field-dependent parameter

$$\Theta_{LA} = \frac{i}{\hbar} \int_0^1 d\kappa \int d^4x \bar{c}^\alpha (\partial^\mu A_\mu^\alpha - \eta^\mu A_\mu^\alpha), \quad (41)$$

with the overall sign changed if the opposite convention is adopted for the BRST differential.

The projected FFBRST transformation is

$$\delta A_\mu^\alpha = D_\mu^{\alpha\beta} c^\beta \Theta_{LA}, \quad (42)$$

$$\delta c^\alpha = -\frac{g}{2} f^{\alpha\beta\gamma} c^\beta c^\gamma \Theta_{LA}, \quad (43)$$

$$\delta \bar{c}^\alpha = B^\alpha \Theta_{LA}, \quad (44)$$

$$\delta B^\alpha = 0. \quad (45)$$

The essential point is that the parameter is not guessed independently: its field content is read directly from the difference of the endpoint gauge-fixing fermions.

Endpoint relation

The BV transgression identity gives

$$\ln \mathcal{J}_{LA} = \frac{i}{\hbar} (S_A - S_L), \quad (46)$$

where S_A is obtained by restricting the same master action to $\mathcal{L}_{\Psi_A}$. Hence

$$\mathcal{J}_{LA} \exp\left(\frac{i}{\hbar} S_L\right) = \exp\left(\frac{i}{\hbar} S_A\right). \quad (47)$$

Therefore,

$$Z_L = \int D\phi e^{iS_L/\hbar} \rightarrow \int D\phi' e^{iS_A/\hbar} = Z_A \quad (48)$$

In the axial endpoint,

$$S_A = S_{YM} + \int d^4x \left[B^\alpha \eta^\mu A_\mu^\alpha + \frac{\lambda}{2} (B^\alpha)^2 - \bar{c}^\alpha \eta^\mu D_\mu^{\alpha\beta} c^\beta \right]. \quad (49)$$

The ghost sector is particularly simple in the usual axial setting, subject to the well-known qualifications concerning residual gauge freedom and spurious poles.

Reducible Abelian two-form theory

Classical gauge structure

Consider an Abelian antisymmetric tensor field $B_{\rho\mu}$ with curvature

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}, \quad (50)$$

and classical action

$$S_0 = \frac{1}{12} \int d^4x H_{\mu\nu\rho} H^{\mu\nu\rho}. \quad (51)$$

The gauge transformation is

$$\delta B_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu. \quad (52)$$

It is reducible because

$$\Lambda_\mu = \partial_\mu \omega \quad (53)$$

produces a vanishing transformation of $B_{\rho\mu}$. Therefore, a complete BV description necessarily contains a ghost-for-ghost.

Minimal BV master action

Introduce the vector ghost ρ_μ and scalar ghost-for-ghost σ . Their antifields are $B^{*\mu\nu}$, $\rho^{*\mu}$ and σ^* . A minimal proper solution is

$$S_{\min}^{(2)} = S_0 + \int d^4x [B^{*\mu\nu} (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu) + \rho^{*\mu} \partial_\mu \sigma], \quad (54)$$

up to conventional factors of i and signs associated with the ghost grading. The classical master equation,

$$(S_{\min}^{(2)}, S_{\min}^{(2)}) = 0, \quad (55)$$

encodes both gauge invariance and reducibility.

Non-minimal sector

For a gauge-fixed formulation, one introduces the non-minimal fields

$$(\tilde{\rho}_\mu, \beta_\mu), \quad (\tilde{\sigma}, \tilde{\chi}), \quad (\varphi, \chi), \quad (56)$$

together with the corresponding antifields. The non-minimal BV action can be written schematically as

$$S_{nm}^{(2)} = \int d^4x [\tilde{\rho}^{*\mu} \beta_\mu + \tilde{\sigma}^* \tilde{\chi} + \varphi^* \chi], \quad (57)$$

with convention-dependent factors of i .

The complete proper solution is

$$S^{(2)} = S_{\min}^{(2)} + S_{nm}^{(2)}. \quad (58)$$

Lorentz-type gauge fermion

A convenient gauge-fixing fermion reproducing the usual Lorentz-type Faddeev-Popov action is

$$\Psi_L^{(2)} = \int d^4x [\tilde{\rho}_\nu (\partial_\mu B^{\mu\nu} - \partial^\nu \varphi + \frac{\lambda_1}{2} \beta^\nu) + \tilde{\sigma} (\partial_\mu \rho^\mu - \frac{\lambda_2}{2} \tilde{\chi})]. \quad (59)$$

Substitution of the antifields determined by

$$\Phi_A^* = \frac{\delta \Psi_L^{(2)}}{\delta \Phi^A} \quad (60)$$

produces the gauge-fixed action

$$\begin{aligned} S_L^{(2)} = \int d^4x [& \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - i \partial_\mu \tilde{\rho}_\nu (\partial^\mu \rho^\nu - \partial^\nu \rho^\mu) + \partial_\mu \tilde{\sigma} \partial^\mu \sigma \\ & + \beta_\nu (\partial_\mu B^{\mu\nu} + \lambda_1 \beta^\nu - \partial^\nu \varphi) - i \tilde{\chi} \partial_\mu \rho^\mu \\ & - i \chi (\partial_\mu \tilde{\rho}^\mu - \lambda_2 \tilde{\chi})], \end{aligned} \quad (61)$$

where the normalization of the auxiliary terms can be shifted by a redefinition of λ_1, λ_2 .

Axial-type gauge fermion

Replace the gauge-fixing derivative by the fixed vector η_μ :

$$\Psi_A^{(2)} = \int d^4x [\tilde{\rho}_\nu (\eta_\mu B^{\mu\nu} + \eta^\nu \varphi + \frac{\lambda_1}{2} \beta^\nu) + \tilde{\sigma} (\eta_\mu \rho^\mu - \frac{\lambda_2}{2} \tilde{\chi})]. \quad (62)$$

The interpolating fermion is

$$\Psi_\kappa^{(2)} = (1 - \kappa) \Psi_L^{(2)} + \kappa \Psi_A^{(2)}. \quad (63)$$

Its tangent is

$$\Psi_A^{(2)} = \int d^4x [\tilde{\rho}_\nu (\eta_\mu B^{\mu\nu} + \eta^\nu \varphi + \frac{\lambda_1}{2} \beta^\nu) + \tilde{\sigma} (\eta_\mu \rho^\mu - \frac{\lambda_2}{2} \tilde{\chi})]. \quad (64)$$

BV-induced finite parameter

The corresponding field-dependent parameter is chosen as the integrated gauge-space tangent:

$$\begin{aligned} \Theta_{LA}^{(2)} = & -\frac{1}{\hbar} \int_0^1 d\kappa \int d^4x [\tilde{\rho}_\nu (\partial_\mu B^{\mu\nu} - \eta_\mu B^{\mu\nu} - \partial^\nu \varphi - \eta^\nu \varphi) \\ & + \tilde{\sigma} (\partial_\mu \rho^\mu - \eta_\mu \rho^\mu)], \end{aligned} \quad (65)$$

again with conventional factors of i depending on the definition of the BRST differential.

The FFBRST projection of the BV flow is

$$\delta B_{\mu\nu} = -(\partial_\mu \rho_\nu - \partial_\nu \rho_\mu) \Theta_{LA}^{(2)}, \quad (66)$$

$$\delta \rho_\mu = -i \partial_\mu \sigma \Theta_{LA}^{(2)}, \quad (67)$$

$$\delta \tilde{\rho}_\mu = i \beta_\mu \Theta_{LA}^{(2)}, \quad (68)$$

$$\delta \tilde{\sigma} = -\tilde{\chi} \Theta_{LA}^{(2)}, \quad (69)$$

$$\delta \varphi = -\chi \Theta_{LA}^{(2)}, \quad (70)$$

with the remaining BRST variations vanishing in the chosen convention.

Endpoint action

The BV transgression relation gives

$$\ln \mathcal{J}_{LA}^{(2)} = \frac{i}{\hbar} [S_A^{(2)} - S_L^{(2)}]. \quad (71)$$

Therefore,

$$\mathcal{J}_{LA}^{(2)} e^{iS_L^{(2)}/\hbar} = e^{iS_A^{(2)}/\hbar}. \quad (72)$$

The axial endpoint is

$$\begin{aligned} S_A^{(2)} = \int d^4x [& \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + i\tilde{\rho}_\nu \eta_\mu (\partial^\mu \rho^\nu - \partial^\nu \rho^\mu) - \tilde{\sigma} \eta_\mu \partial^\mu \sigma \\ & + \beta_\nu (\eta_\mu B^{\mu\nu} + \lambda_1 \beta^\nu + \eta^\nu \varphi) - i\tilde{\chi} \eta_\mu \rho^\mu \\ & - i\chi (\eta_\mu \tilde{\rho}^\mu - \lambda_2 \tilde{\chi})], \end{aligned} \quad (73)$$

which is the axial-type gauge-fixed theory in the convention of the starting action.

Consequently,

$$Z_L^{(2)} = \int D\Phi e^{iS_L^{(2)}/\hbar} \rightarrow \int D\Phi' e^{iS_A^{(2)}/\hbar} = Z_A^{(2)}. \quad (74)$$

The important BV observation is that the ghost and ghost-of-ghost contributions have not been appended independently to the FFBRST construction. They arise simultaneously from the single gauge-fixing fermion and its corresponding Lagrangian submanifold.

General -form and reducible theories

The two-form example illustrates a general mechanism. For an Abelian p -form field $A_{[p]}$,

$$\delta A_{[p]} = d\Lambda_{[p-1]}, \quad (75)$$

there is a reducibility chain

$$\Lambda_{[p-1]}: \Lambda_{[p-1]} + d\Lambda_{[p-2]}: \cdots. \quad (76)$$

The BV master action contains the complete tower of ghosts and ghosts-for-ghosts. Let Ψ_0 and Ψ_1 be two admissible gauge-fixing fermions containing all levels of this hierarchy. Define

$$\Psi_\kappa = (1 - \kappa)\Psi_0 + \kappa\Psi_1. \quad (77)$$

Then

$$\dot{\Psi}_\kappa = \Psi_1 - \Psi_0, \quad (78)$$

and the associated finite field-dependent parameter is obtained by projecting the corresponding BV canonical flow.

The endpoint identity remains

$$\ln \mathcal{J}_{01} = \frac{i}{\hbar} (W_{\Psi_1} - W_{\Psi_0}). \quad (79)$$

Thus the complexity of the reducibility tower is moved from a sequence of independent Jacobian calculations into one BV master action.

This is particularly useful for higher-form theories, supergravity sectors and other reducible systems in which the Faddeev-Popov description requires several generations of auxiliary and ghost fields.

Locality, anomalies and obstructions

Locality is not automatic

A central advantage of the BV formulation is that it makes the limitation of a local Jacobian explicit. The formal expression

$$\Delta G \quad (80)$$

contains coincident functional derivatives and is therefore regulator-dependent. A finite field-dependent transformation admits a local compensating action only if the regulated quantum divergence can be expressed as the functional derivative of a local functional.

Therefore,

$$\text{fielddependence} \not\Rightarrow \text{localJacobian}. \quad (81)$$

Locality must be demonstrated for the specific theory, measure and transformation.

Nonlocality and physical observables

The possible nonlocality of the compensating Jacobian does not by itself imply a nonlocality of physical observables in Yang-Mills theory. The Jacobian characterizes the change of the functional measure associated with a particular field-dependent transformation and may depend on the chosen representation, generator, and regularization prescription. Physical observables are instead identified through the BRST/BV cohomology and are required to be insensitive to admissible changes of gauge fixing. Consequently, as long as the quantum master equation is preserved and the interpolation remains within the admissible gauge-fixing domain, the gauge-independent physical sector is unchanged even when the intermediate compensating functional or its Jacobian does not admit a local representation. In this sense, nonlocality is a property of the intermediate gauge-transport description rather than, by itself, an indication of nonlocal physical observables.

Quantum master equation

If the quantum master equation is violated,

$$\frac{1}{2}(W, W) - i\hbar\Delta W \neq 0, \quad (82)$$

the gauge-independence argument fails. The obstruction is precisely a quantum anomaly in the BV language. Hence the present construction provides a natural diagnostic: the existence of a consistent gauge-space transport requires the quantum master equation to hold in the chosen regularization.

Path dependence

For two admissible gauge-fixing fermions χ and χ' , the partition function is expected to be path independent provided the entire path remains in the same admissible region of gauge-fixing space. Different paths can nevertheless give different parameterizations of the finite transformation. The endpoint physics is unchanged, while the intermediate FFBRST flow is generally not unique.

This non-uniqueness is not a defect. It reflects the freedom to choose different coordinates on the space of gauge-fixing paths.

Gribov regions

The construction is local in gauge-fixing space. If an interpolation crosses a Gribov horizon or encounters a singular gauge condition, the naive continuous flow may cease to be globally valid. In such cases, the relevant integration domain and the BV measure must be restricted or modified.

Consequently, an FFBRST transformation should not be interpreted as an automatic proof of equivalence between arbitrary nonperturbative gauge domains. It proves equivalence only under the assumptions used in defining the path integral and the admissible gauge-fixing family.

Relation to conventional FFBRST and BRST-anti-BRST methods

The present construction is closely connected with the standard FFBRST formalism. The ordinary FFBRST method introduces a continuous parameter κ , computes

$$\frac{d\phi}{d\kappa} = s\phi \Theta_\kappa, \quad (83)$$

and evaluates the corresponding functional Jacobian. The present method keeps the same continuous-flow idea but changes the starting point: the gauge-fixing fermion path is specified first.

The resulting hierarchy is

$$\Psi_0 \rightarrow \Psi_\kappa \rightarrow \Psi_1 \quad (84)$$

in gauge-fixing space,

$$L_{\Psi_0} \rightarrow L_{\Psi_\kappa} \rightarrow L_{\Psi_1} \quad (85)$$

in BV space, and

$$\phi(0) \rightarrow \phi(\kappa) \rightarrow \phi(1) \quad (86)$$

in field space.

The Jacobian is then the quantum measure contribution associated with this transport.

The construction is also distinct from finite BRST-anti-BRST transformations. BRST-anti-BRST methods introduce two nilpotent generators and can accommodate a richer parameter dependence. Here there is one BRST generator, while the antifield sector supplies the geometric information needed to describe changes of gauge.

The distinction can be summarized as:

Table 1. Summary of primary objects and gauge changes across different formulations

Framework	Primary object	Gauge change
BRST	nilpotent differential	BRST-exact deformation
FFBRST	field-dependent parameter	Jacobian-induced action change
BRST-anti-BRST	two generators	extended canonical structure
BV-flow formulation	gauge-fixing fermion path	quantum canonical transport

The new viewpoint is therefore not a new replacement for BV quantization. Rather, it is a reorganization of finite field-dependent gauge changes inside the BV geometry.

Further consequences and extensions*Background-field gauges*

The same formalism can connect ordinary and background-field gauges. The background dependence is incorporated into the gauge-fixing fermion, and the path in can be chosen to preserve background covariance at every intermediate value of .

Higher-spin and higher-form systems

For higher-form theories, the BV complex automatically contains the complete reducibility tower. This suggests that the gauge-space flow may be more efficient than direct component-by-component FFBRST calculations.

Nonlinear gauges

Nothing in the construction requires a linear gauge condition. For

$$G_0[\phi] \quad \text{and} \quad G_1[\phi] \quad (87)$$

nonlinear in the fields, one may define

$$\Psi_\kappa = \Psi[G_\kappa], \quad (88)$$

provided the resulting family remains admissible. The corresponding field-dependent parameter is then determined by the nonlinear gauge-space tangent.

BV and canonical geometry

The formulation suggests a geometric interpretation of FFBRST:

$$\text{FFBRST} = \text{field} - \text{space projection of a quantum canonical BV flow.} \quad (89)$$

Under this interpretation, the familiar Jacobian is not an independent mystery but the quantum divergence associated with the canonical transport.

This relation may also provide a useful bridge between FFBRST and the broader theory of canonical transformations in odd symplectic geometry.

Conclusions

We have developed a BV-based reformulation of finite field-dependent BRST transformations in which the gauge-fixing fermion is treated as the primary variable defining the transformation path. The finite transformation is interpreted as the field-space projection of a quantum canonical flow in the BV field-antifield space.

The principal result is the transgression relation

$$\ln \mathcal{J}_{01} = \frac{i}{\hbar} (W_{\Psi_1} - W_{\Psi_0}), \quad (90)$$

valid under the assumptions of an anomaly-free quantum master equation, consistent measure regularization, and an admissible interpolation between the two gauge-fixing fermions.

For Yang-Mills theory, the construction connects Lorentz and axial gauges. For the reducible Abelian rank-two antisymmetric tensor theory, it incorporates the ghost and ghost-of-ghost sectors into a single BV master action and connects covariant and axial-type gauges through one gauge-fixing path.

The main conceptual advantage is that the gauge choice, FFBRST parameter, canonical BV flow, and measure Jacobian are no longer independent pieces of the calculation. They are different projections of the same quantum transport problem.

At the same time, the BV formulation makes an important limitation explicit: a finite field-dependent transformation does not guarantee a local Jacobian. Locality depends on the regulated BV divergence and on the specific generator. This distinction is essential for nonperturbative applications.

Several directions deserve further study. A complete treatment of Gribov regions could formulate gauge-space transport on restricted BV submanifolds. The relation with finite BRST-anti-BRST transformations may be studied by embedding the present one-generator flow into an extended anticanonical geometry. Finally, the method can be applied to interacting higher-form systems, supergravity and theories with open or reducible gauge algebras.

The author acknowledges useful developments in BRST, FFBRST and BV quantization that motivate the present reformulation.

The present framework may also be relevant to astrophysical and cosmological applications in which gauge fixing and the treatment of unphysical degrees of freedom play an important role. In particular, the BV-based description of finite gauge changes may provide a systematic tool for studying gauge dependence in perturbative quantum gravity, cosmological perturbation theory, and effective field-theory models involving gravitational and higher-form gauge sectors. The explicit separation between gauge-fixing dependence, BRST symmetry, and the quantum measure may be useful when comparing different gauge choices in gravitational and cosmological calculations. A detailed application of the present construction to gravitational and cosmological gauge systems is therefore a natural direction for future work.

Acknowledgements

The author acknowledges useful developments in BRST, FFBRST and BV quantization that motivate the present reformulation.

Author Contribution

Sudhaker Upadhyay conceived and developed the revised finite field-dependent BRST formulation, performed the analytical calculations, derived the Jacobian of the functional measure, carried out the applications to Yang-Mills and Abelian antisymmetric rank-2 tensor field theories, analyzed and interpreted the results, and wrote and approved the final version of the manuscript.

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**Өрістерге тәуелді параметрі бар ақырлы BRST-түрлендірулер Бэталін-Вильковский
формализміндегі кванттық канондық ағындар**

Аңдатпа. Жұмыста өрістерге тәуелді параметрі бар ақырлы BRST-түрлендірулер Баталин–Вильковскийдің өрістер–антиөрістер кеңістігіндегі кванттық канондық ағындар тұрғысынан қарастырылады. Бастапқы құрылым ретінде Якобиан үшін анзац емес, калибрді бекітудің екі фермиондық функциясының арасындағы жол алынады. Калибрдің өзгеруі BV Лагранждық ішкі көптіктерінің бір параметрлі тұқымдасы арқылы сипатталады, ал өрістерге тәуелді параметрі бар BRST-түрлендіру бекітілген калибрдегі BV мастер-әрекеті тудыратын канондық ағынның өрістер кеңістігіне проекциясы ретінде анықталады. Бұл калибрлік фермиондық функция үшін трансгрессиялық теңдеуге әкеледі, онда Якобиан мен калибрді бекіту фермиондық функциясының вариациясы біртұтас кванттық канондық тасымалдау заңының екі құрамдас бөлігі ретінде көрінеді.

Аномалиясыз және кванттық мастер-теңдеуді қанағаттандыратын теория үшін статистикалық қосындының таңдалған калибрді бекіту жолына тәуелсіз екені көрсетіледі. Коварианттық/Лоренцтік және аксиалдық калибрлер біртұтас BV жолының екі шеткі күйі ретінде қарастырылады. Өрістерге тәуелді параметрі бар ақырлы түрлендірулер үшін ықшам компенсациялық шарт шығарылып, Якобианды қалыптастырудағы BV лапласианының нақты рөлі анықталады. Алынған формализм FFBRST-түрлендірулер, калибрді бекітуді анықтайтын канондық түрлендірулер және кванттық мастер-теңдеу арасындағы байланысты нақтылайды. Сонымен қатар, Якобианның локалдылығын жалпы жағдайда алдын ала қабылдауға болмайтыны көрсетіледі: локалдылық ақырлы түрлендірудің өрістерге тәуелділігінің өзімен емес, таңдалған генератормен және регуляризациялау процедурасымен анықталады.

Түйін сөздер: өрістерге тәуелді параметрі бар ақырлы BRST түрлендірулері; Batalin-Vilkovisky формализмі; калибрді бекіту; кванттық канондық түрлендірулер; BRST-BV симметриясы; Якобиан.

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Конечные BRST-преобразования с полевозависимым параметром как квантовые канонические потоки в формализме Бэтали-Вильковского

Аннотация. В работе конечные BRST-преобразования с полевозависимым параметром рассматриваются в терминах квантовых канонических потоков в пространстве полей–антиполей формализма Бэтали–Вильковского. В качестве исходной конструкции используется не анзац для якобиана, а путь между двумя фермионными функциями фиксации калибровки. Изменение калибровки представляется однопараметрическим семейством лагранжевых подмногообразий BV, тогда как преобразование BRST с полевозависимым параметром отождествляется с проекцией на пространство полей канонического потока, порождаемого мастер-действием BV в фиксированной калибровке. Это приводит к трансгрессионному уравнению для калибровочной фермионной функции, в котором якобиан и вариация фермионной функции фиксации калибровки выступают двумя составляющими единого закона квантового канонического переноса.

Для свободной от аномалий теории, удовлетворяющей квантовому мастер-уравнению, показано, что статистикалық қосынды (статистическая сумма) не зависит от выбранного пути фиксации калибровки. Полученная конструкция применяется к неабелевой теории Янга–Миллса и к приводимой абелевой теории антисимметричного тензорного поля второго ранга. В последнем случае полная иерархия духов и духов высшего порядка естественным образом включается в мастер-действие BV. Ковариантная/лоренцева и аксиальная калибровки рассматриваются как два конечных состояния единого BV-пути. Выведено компактное компенсационное условие для конечных преобразований с полевозависимым параметром и определена точная роль лапласиана BV в формировании якобиана. Полученный формализм проясняет связь между преобразованиями FFBRST, каноническими преобразованиями, задающими фиксацию калибровки, и квантовым мастер-уравнением. Показано также, почему локальность якобиана в общем случае не

может предполагаться априори: локальность является свойством выбранного генератора и процедуры регуляризации, а не самим фактом полевой зависимости конечного преобразования.

Ключевые слова: конечные BRST-преобразования с полевозависимым параметром; формализм Баталина–Вильковского; фиксация калибровки; квантовые канонические преобразования; BRST–BV-симметрия; якобиан.

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