

Modified gravity models for inflation with non-minimal coupling trace of the energy-momentum tensor of the scalar field and the Gauss-Bonnet invariant

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Abstract. The article studies the inflationary properties of a cosmological model with the scalar field containing a non-minimal coupling with a modified Gauss-Bonnet invariant $\beta f(G)f(\phi)$ and the trace of the energy-momentum tensor of the scalar field $\alpha f(\phi)$ T in the framework of a flat, homogeneous and isotropic Friedman-Robertson-Walker Universe. To study the inflationary properties of the model, the chaotic potential, the natural potential, and the Starobinsky potential were selected. Within the framework of the slow-rolling approximation and assuming the coupling constants α and β to be small ($\alpha \ll 1$ and $\beta \ll 1$), generalized expressions for the scalar spectral index n_s and the scalar tensor ratio r are obtained. In the work, the Cobaya cosmology shell for cosmological calculations together with the Polychord sampler was used to calculate the theoretical values of the scalar spectral index and the scalar-tensor ratio and compare them with the observables. It is shown that for the chaotic potential, only the value of the scalar spectral index agrees with the observed data of Planck2018, and the value of the scalar-tensor ratio r exceeds the observed value. For the natural potential and the Starobinsky potential, both values of the inflationary parameters are consistent with observations. It is shown that for the case $f(G)=G$ and all types of potentials, the coupling parameter α is marginalized in each model, and the coupling parameter β does not affect the dynamics of inflationary parameters and is not constrained in any way.

Keywords: inflation, Gauss-Bonnet term, energy-momentum tensor, slow roll approximation, scalar spectral index, scalar-to-tensor ratio, observation constraint.

Received 14.06.2026. Revised 14.08.2026. Accepted 15.08.2026. Available online 30.09.2026

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Introduction

Modern cosmology is based on Einstein's General Theory of Relativity and assumes that on scales larger than galaxies, the Universe is homogeneous and isotropic. This model is described by the Friedmann–Robertson–Walker metric, in which the scale factor determines the degree of the Universe's expansion over time. The evolution of this factor is governed by the Friedmann equations, which relate the expansion rate to the energy density and pressure of various components (radiation, matter, dark energy). However, the standard Big Bang model faces several fundamental problems. The flatness problem arises from the fact that even tiny deviations from the critical density in the early Universe would have led to a highly curved geometry today – which we do not observe. The horizon problem stems from the observation that opposite regions of the Universe, such as those seen in the cosmic microwave background, could not have been in causal contact – yet they exhibit the same temperature and properties. These paradoxes are resolved by the introduction of the inflationary theory, according to which, in the first fractions of a second after its birth, the Universe underwent a phase of exponential expansion. As a result, even very small regions of space were “stretched” to cosmological scales, ensuring uniform physical conditions and smoothing out the curvature. Thus, inflation simultaneously solves both problems: it makes the Universe appear flat and establishes a causal connection between distant regions [1]. Inflationary cosmology also provides the initial conditions for the formation of large-scale structures, explaining the origin of density fluctuations that later evolved into galaxies and clusters. These quantum fluctuations, which arose during the inflationary period, became “frozen” into the fabric of spacetime and are observed today as anisotropies in the cosmic microwave background radiation (for reviews, see, e.g., Refs. [2, 3]). At present, inflation is considered an integral part of the standard cosmological model. However, the exact mechanisms underlying the inflationary expansion of the Universe remain not fully understood. There are numerous competing inflationary models, among which single-field slow-roll models attract particular interest due to their structural simplicity and effectiveness. Nevertheless, recent data from the Planck mission in 2018 [4] have cast doubt on the validity of some of these models. Before moving on to more complex constructions of inflationary scenarios, the authors suggest approaching the problem from an alternative perspective.

Modern modified theories of gravity, such as $f(R)$ gravity and its extensions, offer a promising approach to providing a unified description of both early-time inflation and the late-time accelerated expansion of the Universe – without the need to introduce dark energy as a separate component. Within these theories, the geometric part of the gravitational action is modified in such a way that cosmic acceleration arises naturally from the structure of spacetime itself. This not only allows for the reproduction of inflationary dynamics but also offers an explanation for phenomena traditionally attributed to dark matter and dark energy. Thus, modified gravity provides an opportunity to reconsider the standard Λ CDM model by addressing its conceptual challenges, including the origin and the extremely small value of the cosmological constant [5]. The Λ CDM model, which incorporates the cosmological constant Λ , provides a successful fit to observational data and is therefore regarded as the standard model of cosmology. However, it does not offer a complete explanation of cosmological dynamics, as the two dominant components—dark energy and dark matter—remain insufficiently understood. In particular, the physical or geometrical origin of the cosmological constant Λ , as well as its remarkably small value, still requires clarification. By moving beyond the assumption that the Universe is precisely described by the Λ CDM framework, one can consider modified gravity theories, which are extensions of Einstein's general relativity, to address these unresolved issues [6]. These extensions may involve modifications to the geometrical part, the matter part, or both components of the Einstein–Hilbert action. Numerous modified gravity models have been developed, such as $f(R)$ gravity [7] and $f(T)$ gravity [8], where T denotes the torsion scalar in the teleparallel formulation. Moreover, theories with non-minimal coupling between geometry and matter have also been explored, including $f(R, L_m)$ gravity [9] and $f(R, T)$ gravity [10], where L_m and T are the matter Lagrangian and the trace of the energy–momentum tensor, respectively. One of the main motivations for considering such modifications is the possibility of naturally explaining the late-time acceleration of the Universe without introducing a separate dark energy component [11].

Beyond late-time cosmology, modified gravity models are also relevant in the context of the early Universe, particularly during the inflationary epoch. From a phenomenological standpoint, these theories may serve as effective descriptions of a more fundamental, yet unknown, quantum theory of gravity below a certain energy scale. Within this perspective, it is plausible that such modifications leave observable signatures during the inflationary phase.

For instance, Higgs inflation is based on a non-minimal coupling to the Ricci scalar, while the Starobinsky model [12,13] incorporates a quadratic R^2 term in the action and is known for its excellent agreement with observational data when expressed in the Einstein frame. The interplay between inflation and modified gravity has been widely discussed in the literature [14-23], with a comprehensive overview presented in [24].

In this work, we propose a novel modified gravity model, referred to as $f(\varphi)T$ gravity. This theory extends Einstein-Gauss-Bonnet gravity [25] by introducing a direct coupling between the inflaton field φ and the trace T of the energy-momentum tensor. The action of $f(\varphi)T$ gravity is

$$S = \int \left(\frac{R}{2\kappa} + \beta f(G)f(\varphi) + \alpha f(\varphi)T + \mathcal{L}_m \right) \sqrt{-g} d^4x \quad (1)$$

$$\equiv 8\pi G \equiv \frac{1}{M_{Pl}^2}, M_{Pl} = 1$$

where $\kappa \equiv 8\pi G \equiv \frac{1}{M_{Pl}^2}$, $M_{Pl} = 1$, is the reduced Planck mass and R is the Ricci scalar, T is the trace of the energy-momentum tensor, $\mathcal{L}_m$ is the matter Lagrangian.

The function f is a dimensionless function of the inflaton φ , satisfying the condition $f(0)=0$, ensuring the recovery of General Relativity (GR) in the absence of the inflaton. Thus, this model represents a modification of gravity that operates exclusively during the inflationary period. In the limit $\alpha \rightarrow 0$, the theory reduces to standard GR.

In this work, we consider the simplest case

$$f(\varphi) = \sqrt{\kappa}\varphi \quad (2)$$

This approach generalizes the form of modified gravity

$$f(R, T) = R + 2\kappa\alpha T \quad (3)$$

interpreting α as a field identified with the inflaton. Such a modification of gravity has not been explored previously and can be motivated within certain semiclassical frameworks of quantum gravity [26, 27].

General framework

We introduce a new model of $f(\varphi)T$ - gravity, an extension of GR with a direct inflaton – trace coupling of the energy – momentum tensor. The action of $f(\varphi)T$ - gravity has the following form

$$S = \int \left(\frac{R}{2\kappa} + \beta f(G)f(\varphi) + \alpha f(\varphi)T + \mathcal{L}_m \right) \sqrt{-g} d^4x \quad (4)$$

We consider action (4) together Friedman-Robertson-Walker metric describes spatial flat and homogeneous space-time $ds^2 = dt^2 + a^2(t)dx_i dx^i$. Here we give notations for our action such as the Ricci scalar and the Gauss-Bonnet invariant, matter Lagrangian and trace of the energy-momentum tensor

$$G = 24 \frac{\ddot{a}\dot{a}^2}{a^3}, R = 6 \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right), \mathcal{L}_m = \frac{1}{2} \dot{\varphi}^2 - V(\varphi), T = \dot{\varphi}^2 - 4V(\varphi) \quad (5)$$

Taking into account the FRW metric, we rewrite the action, substitute R and introducing indefinite Lagrange multiplier

$$S = \int dt a^3 \left[\frac{3}{k} \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) + \beta \sqrt{k} f(G) \varphi + \alpha \sqrt{k} \varphi \dot{\varphi}^2 - 4\alpha \sqrt{k} \varphi V(\varphi) + \frac{1}{2} \dot{\varphi}^2 - V(\varphi) \right]$$

$$\lambda \left(G - 24 \frac{\ddot{a}\dot{a}^2}{a^3} \right) \quad (6)$$

To find the indefinite Lagrange multiplier, it is necessary to differentiate the action by G and equate the derivative to zero, which corresponds to the extremum condition

$$\frac{dS}{dG} = 0 \Rightarrow [\beta\sqrt{\kappa}f_G(G)\varphi a^3 - \lambda a^3] = 0 \Rightarrow \lambda = \beta\sqrt{\kappa}f_G(G)\varphi \quad (7)$$

We obtain lagrangian, after a simple transformation

$$L = -\frac{3}{\kappa}\dot{a}^2 a + \beta\sqrt{\kappa}f(G)a^3\dot{\varphi} - \beta\sqrt{\kappa}f_G G a^3\dot{\varphi} + \alpha\sqrt{\kappa}\dot{\varphi}^2 a^3\dot{\varphi} - 4\alpha\sqrt{\kappa}V(\varphi)a^3\dot{\varphi} + \frac{1}{2}\dot{\varphi}^2 a^3 - V(\varphi)a^3 - 8\beta\sqrt{\kappa}f_G a^3\dot{\varphi} - 8\beta\sqrt{\kappa}f_{GG} G a^3\dot{\varphi} \\ \frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0, \quad (8)$$

Applying the Euler - Lagrange equation $\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)$, for the scale factor $a(t)$, we obtain

$$\frac{2}{\kappa}\ddot{H} + \frac{3}{\kappa}H^2 = -\frac{1}{2}\dot{\varphi}^2 + V(\varphi) - \beta\sqrt{\kappa}f(G)\dot{\varphi} - \alpha\sqrt{\kappa}\dot{\varphi}^2\dot{\varphi} + 4\alpha\sqrt{\kappa}\varphi V(\varphi) + \beta\sqrt{\kappa}f_G G\dot{\varphi} \\ - 8\beta\sqrt{\kappa}f_G \dot{\varphi} H^2 - 16\beta\sqrt{\kappa}f_{GG} G \dot{\varphi} H^2 - 16\beta\sqrt{\kappa}f_G \dot{\varphi} H\ddot{H} - 16\beta\sqrt{\kappa}f_G \dot{\varphi} H^3 \\ - 8\beta\sqrt{\kappa}f_{GGG} G^2 H^2\dot{\varphi} - 8\beta\sqrt{\kappa}f_{GG} G H^2\dot{\varphi} - 16\beta\sqrt{\kappa}f_{GG} G H\ddot{H}\dot{\varphi} - \\ 16\beta\sqrt{\kappa}f_{GG} G H^3\dot{\varphi} \quad (9)$$

Then we find the total energy $\frac{\partial L}{\partial a}\dot{a} + \frac{\partial L}{\partial \dot{\varphi}}\dot{\varphi} + \frac{\partial L}{\partial G}\dot{G} - L = 0$, we obtain

$$\frac{\partial L}{\partial a}\dot{a} + \frac{\partial L}{\partial \dot{\varphi}}\dot{\varphi} + \frac{\partial L}{\partial G}\dot{G} - L = 0, \\ \frac{3}{\kappa}H^2 = -24\beta\sqrt{\kappa}f_G \dot{\varphi} H^3 - 24\beta\sqrt{\kappa}f_{GG} G H^3\dot{\varphi} - \beta\sqrt{\kappa}f(G)\dot{\varphi} + \frac{\dot{\varphi}^2}{2}(1 + 2\alpha\sqrt{\kappa}\varphi) \\ + V(\varphi)(1 + 4\alpha\sqrt{\kappa}\varphi) + \beta\sqrt{\kappa}f_G G\dot{\varphi} \\ \frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0 \text{ f} \quad (10)$$

Applying the Euler - Lagrange equation $\frac{\partial L}{\partial q} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0$ for the scalar field $\varphi(t)$, we obtain

$$\ddot{\varphi}(1 + 2\alpha\sqrt{\kappa}\varphi) + 3H\dot{\varphi}(1 + 2\alpha\sqrt{\kappa}\varphi) - \alpha\sqrt{\kappa}\dot{\varphi}^2 + V'(\varphi)(1 + 4\alpha\sqrt{\kappa}\varphi) \\ + 4\alpha\sqrt{\kappa}V(\varphi) - \beta\sqrt{\kappa}f(G) = 0 \quad (11)$$

In the below parts we consider $f(G)=G$. That reduction is physically existing case. In that case equations (10)-(11) take the form

$$\frac{2}{\kappa}\ddot{H} + \frac{3}{\kappa}H^2 = -\frac{\dot{\varphi}^2}{2}(1 + 2\alpha\sqrt{\kappa}\varphi) + V(\varphi)(1 + 4\alpha\sqrt{\kappa}\varphi) - 8\beta\sqrt{\kappa}\dot{\varphi} H^2 - \\ 16\beta\sqrt{\kappa}\dot{\varphi} H(H + H^2) \quad (9)(12)$$

$$\begin{aligned} \frac{3}{\kappa} H^2 = & -24\beta\sqrt{\kappa}\dot{\varphi}H^3 + \frac{\ddot{\varphi}^2}{2}(1 + 2\alpha\sqrt{\kappa}\varphi) + V(\varphi)(1 + 4\alpha\sqrt{\kappa}\varphi) \\ & \ddot{\varphi}(1 + 2\alpha\sqrt{\kappa}\varphi) + 3H\dot{\varphi}(1 + 2\alpha\sqrt{\kappa}\varphi) - \alpha\sqrt{\kappa}\dot{\varphi}^2 + V'(\varphi)(1 + 4\alpha\sqrt{\kappa}\varphi) + \\ & 4\alpha\sqrt{\kappa}V(\varphi) \end{aligned} \quad (12)$$

$$-24\beta\sqrt{\kappa}H^2 \left(\dot{H} + H^2 \right) = 0 \quad (13)$$

The methodology. Slow roll approximation in the nonminimal coupling model

The slow roll approximation of the scalar field φ with respect to the potential $V(\varphi)$ is determined by the conditions

$$\frac{\dot{\varphi}^2}{V} \ll 1, \left| \frac{\ddot{\varphi}}{H\dot{\varphi}} \right| \ll 1, \frac{\sqrt{\kappa}\dot{\varphi}^2}{H\varphi} \ll 1 \quad (14)$$

Applying them to equations (12)-(13), we obtain

$$\frac{3}{\kappa} H^2 \cong V(\varphi)(1 + 4\alpha\sqrt{\kappa}\varphi) - 24\beta\sqrt{\kappa}\dot{\varphi}H^3 \quad (15)$$

$$3H\dot{\varphi}(1 + 2\alpha\sqrt{\kappa}\varphi) + V'(\varphi)(1 + 4\alpha\sqrt{\kappa}\varphi) + 4\alpha\sqrt{\kappa}V(\varphi) - 24\beta\sqrt{\kappa}H^4 \cong 0. \quad (16)$$

Then the effective potential and its derivative are determined by the following expressions

$$V^{eff} = V(\varphi)(1 + 4\alpha\sqrt{\kappa}\varphi) \quad (17)$$

$$V_{\varphi}^{eff} = \frac{V'(\varphi)(1+4\alpha\sqrt{\kappa}\varphi) + 4\alpha\sqrt{\kappa}V(\varphi) - \frac{8\beta\kappa^{\frac{5}{2}}}{3}V^2(\varphi)(1+4\alpha\sqrt{\kappa}\varphi)^2}{1+2\alpha\sqrt{\kappa}\varphi} \quad (18)$$

Let's consider equations (15) and (16) as a system equation and assume that $\alpha \ll 1$ and $\beta \ll 1$ and omit the higher-order terms, we can find that

$$H^2 = \frac{1}{3}\kappa V(\varphi) + \frac{4}{3}\alpha\kappa^{\frac{3}{2}}V(\varphi) + \frac{8}{9}\beta\kappa^{\frac{5}{2}}V(\varphi)V'(\varphi) + O(\alpha^2) + O(\beta^2) + O(\alpha\beta), \quad (19)$$

The Hubble slow roll parameters ϵ_H and η_H are expressed in terms of the effective potential and its derivatives V^{eff} , V_{φ}^{eff} and $V_{\varphi\varphi}^{eff}$

$$\epsilon_H = -\frac{H}{H^2}, \eta_H = \frac{\epsilon_H}{H\epsilon_H} \quad (20)$$

Next, the slow roll parameters (20) are used to calculate the spectral index of scalar perturbations n_s and the tensor-to-scalar ratio r , whose values can be compared with observational data

$$n_s = 1 - 2\epsilon_H - \eta_H, r = 16\epsilon_H \quad (21)$$

In the $f(\varphi)T$ modified gravity, substituting the potential into (15) and (16), we get the slow roll parameters

$$\epsilon_H = \frac{1}{2\kappa} \left(\frac{V'(\varphi)}{V(\varphi)} \right)^2 + \frac{\alpha}{\sqrt{\kappa}} \left(\frac{4V'(\varphi)}{V(\varphi)} - \varphi \left(\frac{V'(\varphi)}{V(\varphi)} \right)^2 \right) + \frac{4}{3} \beta \kappa^{\frac{1}{2}} \left(\frac{V'(\varphi)V''(\varphi)}{V(\varphi)} - \frac{V'^3(\varphi)}{V^2(\varphi)} - \kappa V'(\varphi) \right) \quad (22)$$

$$\eta_H = \frac{1}{\kappa} \frac{V''(\varphi)}{V(\varphi)} + \frac{2\alpha}{\sqrt{\kappa}} \left(\frac{4V'(\varphi)}{V(\varphi)} - \frac{3V''(\varphi)}{V(\varphi)} \varphi \right) + \frac{4}{3} \beta \kappa^{\frac{1}{2}} \left(\frac{4V^2(\varphi)V''(\varphi)}{V'(\varphi)} + \frac{4V'^3(\varphi)}{V^2(\varphi)} - \frac{V'(\varphi)V''(\varphi)}{V(\varphi)} - 2V''(\varphi) + 4\kappa \left(V'(\varphi) + \frac{V(\varphi)V''(\varphi)}{V'(\varphi)} \right) \right) \quad (23)$$

The number of e-folds is

$$\begin{aligned} N &= \int_{\varphi^*}^{\varphi_{end}} \frac{H}{\varphi} d\varphi \\ &= \int_{\varphi_{end}}^{\varphi^*} \frac{(1 + 2\alpha\sqrt{\kappa}\varphi) \left(\kappa V(\varphi) + 4\alpha\kappa^{\frac{3}{2}}\varphi V(\varphi) + \frac{8}{3}\beta\kappa^{\frac{5}{2}}V(\varphi)V'(\varphi) \right)}{V'(\varphi)(1 + 4\alpha\sqrt{\kappa}\varphi) + 4\alpha\sqrt{\kappa}V(\varphi) - 24\beta\sqrt{\kappa}H^2 - \beta\sqrt{\kappa}H^4} d\varphi \\ &= \int_{\varphi_{end}}^{\varphi^*} \left[\frac{\kappa V(\varphi)}{V'(\varphi)} + 2\alpha\kappa^{\frac{3}{2}} \left(\varphi \frac{V(\varphi)}{V'(\varphi)} - 2 \frac{V^2(\varphi)}{V'^2(\varphi)} \right) + 8\beta\kappa^{\frac{5}{2}} \left(\frac{V(\varphi)}{3} + 3\kappa \frac{V^3(\varphi)}{V'^3(\varphi)} \right) \right] d\varphi \end{aligned} \quad (24)$$

We omit the higher-order terms in α and β in the last expression. With the field value evaluated at the horizon crossing using (24), one can calculate the observables n_s and r using (21).

Findings and discussion. Application to inflationary models

1. Chaotic inflation

We consider first the chaotic inflation model [26], where the potential is of the power-law form

$$V = \lambda M_{Pl}^4 \left(\frac{\varphi}{M_{Pl}} \right)^n \quad (25)$$

In the $f(\varphi)T$ modified gravity, substituting the potential into (17) and (18), we get the slow roll parameters

$$\epsilon_H = \frac{n^2}{2\kappa\varphi^2} + \frac{\alpha n}{\sqrt{\kappa}\varphi} (4 - n) + \beta \left(-\frac{4}{3} n^2 \kappa^{\frac{1}{2}} \lambda \varphi^{n-3} - \frac{4}{3} n \kappa^{\frac{3}{2}} \lambda \varphi^{n-1} \right) \quad (26)$$

$$\begin{aligned} \eta_H &= \frac{2n}{\kappa\varphi^2} + \frac{\alpha}{\sqrt{\kappa}\varphi} (8n - 32) + \beta \left(\frac{8}{3} n^2 \kappa^{\frac{1}{2}} \lambda \varphi^{n-3} - \frac{8}{3} n^3 \kappa^{\frac{1}{2}} \lambda \varphi^{n-3} + \right. \\ &\quad \left. \frac{8}{3} n \kappa^{\frac{1}{2}} \lambda \varphi^{n-2} - \frac{8}{3} n^2 \kappa^{\frac{1}{2}} \lambda \varphi^{n-2} - \frac{4}{3} \kappa^{\frac{3}{2}} \lambda \varphi^{n-1} + \frac{20}{3} n \kappa^{\frac{1}{2}} \lambda \varphi^{n-1} + \frac{16}{3} \kappa^{\frac{1}{2}} \lambda \varphi^n - \frac{16}{3n} \kappa^{\frac{1}{2}} \lambda \varphi^n \right) \end{aligned} \quad (27)$$

The spectral index and tensor-to-scalar ratio are modified accordingly to be

$$n_s = 1 - \frac{n^2}{\kappa\varphi^2} - \frac{2n}{\kappa\varphi^2} + \frac{\alpha}{\sqrt{\kappa}\varphi} (32 - 16n + 2n^2) + \beta \left(-\frac{16}{3} n^2 \kappa^{\frac{1}{2}} \lambda \varphi^{n-3} + \frac{8}{3} n^3 \kappa^{\frac{1}{2}} \lambda \varphi^{n-3} - \frac{8}{3} n \kappa^{\frac{1}{2}} \lambda \varphi^{n-2} + \frac{8}{3} n^2 \kappa^{\frac{1}{2}} \lambda \varphi^{n-2} + \frac{4}{3} \kappa^{\frac{3}{2}} \lambda \varphi^{n-1} - 4n \kappa^{\frac{3}{2}} \lambda \varphi^{n-1} - \frac{16}{3} \kappa^{\frac{1}{2}} \lambda \varphi^n + \frac{16}{3n} \kappa^{\frac{1}{2}} \lambda \varphi^n \right) \quad (28)$$

$$r = \frac{8n^2}{\kappa\varphi^2} + \frac{16\alpha n}{\sqrt{\kappa}\varphi} (4 - n) + \beta \left(-\frac{64}{3} n^2 \kappa^{\frac{1}{2}} \lambda \varphi^{n-3} - \frac{64}{3} n \kappa^{\frac{3}{2}} \lambda \varphi^{n-1} \right) \quad (29)$$

The spectral index and tensor-to-scalar ratio are calculated by (17) and their MCMC corner plot present in Figure 1. We have three free parameters α , β and n . The other internal parameters are expressed in terms of the free parameters. Table 1 shows the mean values of the coupling constants α and β for reconstruction with chaotic potential.

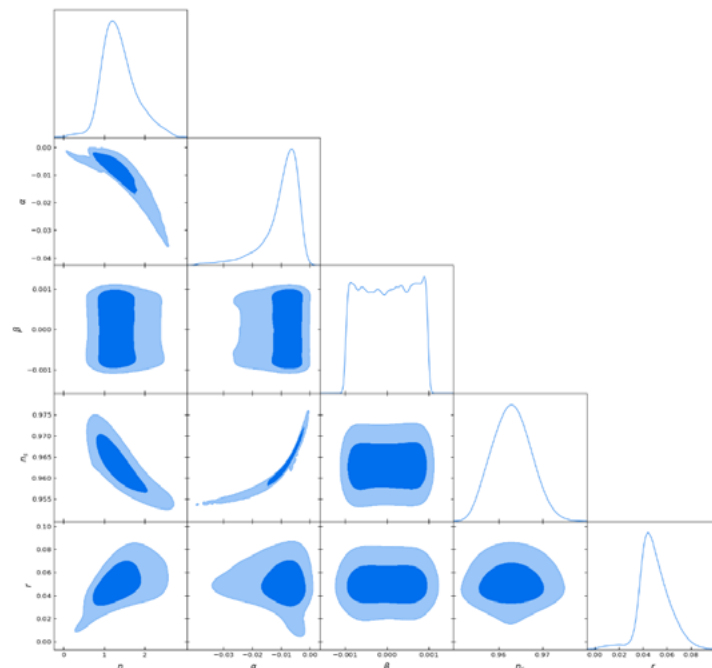


Figure 1. The spectral index and tensor-to-scalar ratio for the chaotic inflation potential (25). The range of α , β and n is Marginalized joint 68% and 95% C.L. regions for α and β at from Planck 2018 data [4] are shown in blue and light blue, respectively.

Table 1. Parameter values in the confidence interval for the chaotic inflation potential (25).

α	β	n	n_s	r
-0.0098 ± 0.0063	No limit	1.38 ± 0.42	0.96295 ± 0.00434	0.050 ± 0.01

2. Natural inflation

Natural inflation [27] is a well-motivated inflation model, as the flatness of the inflationary potential required by the slow-roll is guaranteed by a quasishift symmetry. In natural inflation, the inflation is an axionlike particle and has a potential of the following form

$$V = \Lambda^4 \left[1 + \cos\left(\frac{\varphi}{f}\right) \right] \quad (30)$$

Within the nonminimal coupling gravity, the slow-roll parameters are

$$\epsilon_H = \frac{\sin^2\left(\frac{\varphi}{f}\right)}{2f\kappa\left(1+\cos\left(\frac{\varphi}{f}\right)\right)^2} - \alpha \left(\frac{4\sin\left(\frac{\varphi}{f}\right)}{f\kappa^{\frac{1}{2}}\left(1+\cos\left(\frac{\varphi}{f}\right)\right)} + \frac{\varphi\sin^2\left(\frac{\varphi}{f}\right)}{f^2\kappa^{\frac{1}{2}}\left(1+\cos\left(\frac{\varphi}{f}\right)\right)^2} \right) + \beta \left(\frac{4\kappa^{\frac{3}{2}}\Lambda^4\sin\left(\frac{\varphi}{f}\right)}{3f} + \frac{4\kappa^{\frac{1}{2}}\Lambda^4\cos\left(\frac{\varphi}{f}\right)\sin\left(\frac{\varphi}{f}\right)}{3f^3\left(1+\cos\left(\frac{\varphi}{f}\right)\right)} + \frac{4\kappa^{\frac{1}{2}}\Lambda^4\sin^3\left(\frac{\varphi}{f}\right)}{3f^3\left(1+\cos\left(\frac{\varphi}{f}\right)\right)^2} \right) \quad (31)$$

$$\eta_H = -\frac{2\cos\left(\frac{\varphi}{f}\right)}{f^2\kappa\left(1+\cos\left(\frac{\varphi}{f}\right)\right)} + \frac{2\sin^2\left(\frac{\varphi}{f}\right)}{f^2\kappa\left(1+\cos\left(\frac{\varphi}{f}\right)\right)^2} - \frac{4\alpha\left(\frac{8ctg\left[\frac{\varphi}{f}\right]}{f} + \frac{6\sin\left[\frac{\varphi}{f}\right]}{f\left(1+\cos\left[\frac{\varphi}{f}\right]\right)}\right)}{\sqrt{\kappa}} + \frac{4}{3}\beta\sqrt{\kappa}\left(\frac{2\Lambda^4\cos\left[\frac{\varphi}{f}\right]}{f^2} - 4\Lambda^4\left(1+\cos\left[\frac{\varphi}{f}\right]\right)^2ctg\left[\frac{\varphi}{f}\right]csc\left[\frac{\varphi}{f}\right] - \frac{6\Lambda^4\cos\left[\frac{\varphi}{f}\right]\sin\left[\frac{\varphi}{f}\right]}{f^3\left(1+\cos\left[\frac{\varphi}{f}\right]\right)} - \frac{4\Lambda^4\sin\left[\frac{\varphi}{f}\right]^3}{f^3\left(1+\cos\left[\frac{\varphi}{f}\right]\right)^2} + \kappa\left(\frac{\Lambda^4\left(1+\cos\left[\frac{\varphi}{f}\right]\right)Cot\left[\frac{\varphi}{f}\right]}{f} - \frac{4\Lambda^4\sin\left[\frac{\varphi}{f}\right]}{f}\right)\right) \quad (32)$$

We see again that when $\alpha \rightarrow 0$, the results of $f(\varphi)T$ modified gravity go to the Einstein gravity ones. The spectral index and tensor-to-scalar ratio in the of $f(\varphi)T$ gravity can be expressed accordingly

$$n_s = 1 - \frac{3\sin^2\left(\frac{\varphi}{f}\right)}{f^2\kappa\left(1+\cos\left(\frac{\varphi}{f}\right)\right)^2} - \frac{2\cos\left(\frac{\varphi}{f}\right)}{f^2\kappa\left(1+\cos\left(\frac{\varphi}{f}\right)\right)} - \alpha \left(-\frac{16\sin\left(\frac{\varphi}{f}\right)}{f\kappa^{\frac{1}{2}}\left(1+\cos\left(\frac{\varphi}{f}\right)\right)} + \frac{2\varphi\sin^2\left(\frac{\varphi}{f}\right)}{f^2\kappa^{\frac{1}{2}}\left(1+\cos\left(\frac{\varphi}{f}\right)\right)^2} - \frac{32ctg\left(\frac{\varphi}{f}\right)}{f\kappa^{\frac{1}{2}}} \right) + \beta \left(-\frac{8\kappa^{\frac{1}{2}}\Lambda^4\sin\left(\frac{\varphi}{f}\right)}{3f^2} - \frac{4\kappa^{\frac{3}{2}}\Lambda^4\cos\left(\frac{\varphi}{f}\right)}{3f} - \frac{4\kappa^{\frac{3}{2}}\Lambda^4\cos\left(\frac{\varphi}{f}\right)ctg\left(\frac{\varphi}{f}\right)}{3f} + \frac{32\kappa^{\frac{1}{2}}\Lambda^4ctg^2\left(\frac{\varphi}{f}\right)}{3} + \frac{16\kappa^{\frac{1}{2}}\Lambda^4\cos\left(\frac{\varphi}{f}\right)ctg^2\left(\frac{\varphi}{f}\right)}{3} + \frac{16\kappa^{\frac{1}{2}}\Lambda^4\cos\left(\frac{\varphi}{f}\right)csc\left(\frac{\varphi}{f}\right)}{3} + \frac{8\kappa^{\frac{3}{2}}\Lambda^4\sin\left(\frac{\varphi}{f}\right)}{3f} + \frac{16\kappa^{\frac{1}{2}}\Lambda^4\sin\left(\frac{\varphi}{f}\right)\cos\left(\frac{\varphi}{f}\right)}{3f^3\left(1+\cos\left(\frac{\varphi}{f}\right)\right)} + \frac{8\kappa^{\frac{1}{2}}\Lambda^4\sin^3\left(\frac{\varphi}{f}\right)}{3f^3\left(1+\cos\left(\frac{\varphi}{f}\right)\right)^2} \right) \quad (33)$$

$$r = \frac{8\sin^2\left(\frac{\varphi}{f}\right)}{f^2\kappa\left(1+\cos\left(\frac{\varphi}{f}\right)\right)^2} + \alpha \left(-\frac{64\sin\left(\frac{\varphi}{f}\right)}{f\kappa^{\frac{1}{2}}\left(1+\cos\left(\frac{\varphi}{f}\right)\right)} - \frac{16\varphi\sin^2\left(\frac{\varphi}{f}\right)}{f^2\kappa^{\frac{1}{2}}\left(1+\cos\left(\frac{\varphi}{f}\right)\right)^2} \right) + \beta \left(\frac{64\kappa^{\frac{3}{2}}\Lambda^4\sin\left(\frac{\varphi}{f}\right)}{3f} + \frac{64\kappa^{\frac{1}{2}}\Lambda^4\cos\left(\frac{\varphi}{f}\right)\sin\left(\frac{\varphi}{f}\right)}{3f^3\left(1+\cos\left(\frac{\varphi}{f}\right)\right)} + \frac{64\kappa^{\frac{1}{2}}\Lambda^4\sin^3\left(\frac{\varphi}{f}\right)}{3f^3\left(1+\cos\left(\frac{\varphi}{f}\right)\right)^2} \right) \quad (34)$$

The spectral index and tensor-to-scalar ratio are calculated by (21) and their corner MCMC plot present in Figure 2. We have three free parameters α , β . The other internal parameters are expressed in terms of the free parameters. Table 2 shows the mean values of the coupling constants α and β for reconstruction with natural potential.

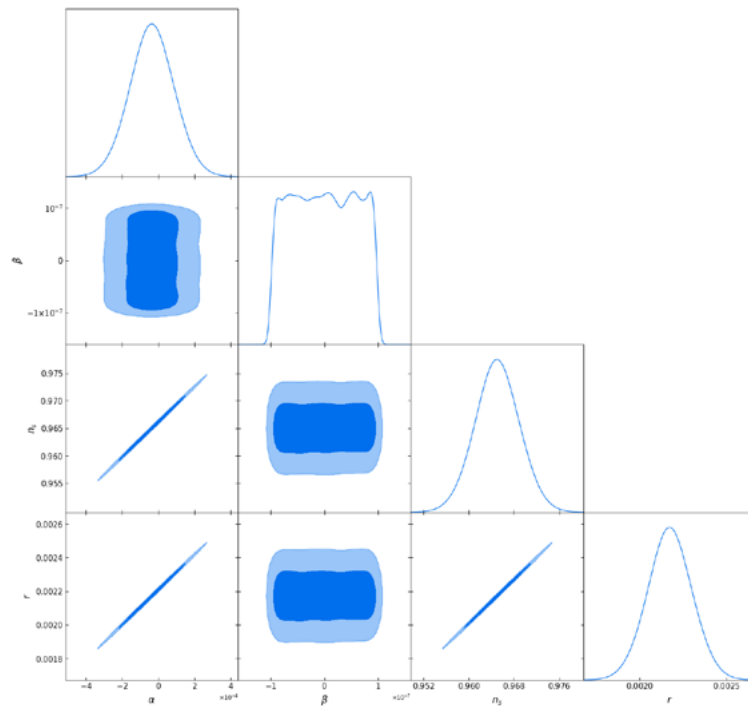


Figure 2. The spectral index and tensor-to-scalar ratio for the natural inflation potential (30). The range of is for , is . Marginalized joint 68% and 95% C.L. regions for and at from Planck 2018 data [4] are shown in blue and light blue, respectively.

Table 2. Parameter values in the confidence interval for the natural inflation potential (30).

α	β	f	n_s	r
$(-1.099 \pm 0.153) \times 10^{-3}$	No limit	4	0.96491 ± 0.00419	0.01327 ± 0.00036

3. Starobinsky inflation

In this subsection, we will consider the effective potential of the Starobinsky inflation [13] in Einstein frame as

$$V = \Lambda^4 \left(1 - e^{-\sqrt{\frac{2\kappa}{3}}\varphi} \right)^2 \quad (35)$$

For Starobinsky inflation we also find the slow roll approximation parameters. To simplify the expression, we will introduce some definitions such as

$$\epsilon_H = \frac{2e^{-2\varphi}}{\kappa(1-e^{-\varphi})^2} + \alpha \left(\frac{8e^{-\varphi}}{\kappa^{\frac{1}{2}}(1-e^{-\varphi})} - \frac{4e^{-2\varphi}\varphi}{\kappa(1-e^{-\varphi})^2} \right) + \beta \left(\frac{8}{3}\kappa^{\frac{3}{2}}\Lambda^4 e^{-2\varphi} - \frac{8}{3}\kappa^{\frac{3}{2}}\Lambda^4 e^{-\varphi} - \frac{16\Lambda^4\kappa^{\frac{1}{2}}e^{-\varphi}}{3(1-e^{-\varphi})} \right) \quad (36)$$

$$\eta_H = \frac{4e^{-\varphi}}{\kappa(1-e^{-\varphi})^2} + \alpha \left(\frac{16e^{-\varphi}}{\kappa^{\frac{1}{2}}(1-e^{-\varphi})} - \frac{32}{\kappa^{\frac{1}{2}}(1-e^{-\varphi})} \right) + \beta \left(\frac{32}{3} \kappa^{\frac{1}{2}} \Lambda^4 - \frac{16}{3} \kappa^{\frac{3}{2}} \Lambda^4 e^{-2\varphi} - \right. \\ \left. 8\kappa^{\frac{1}{2}} \Lambda^4 e^{-\varphi} - \frac{8}{3} \kappa^{\frac{1}{2}} \Lambda^4 e^{\varphi} - \frac{64\kappa^{\frac{1}{2}} \Lambda^4 e^{-3\varphi}}{3(1-e^{-\varphi})} + \frac{32\kappa^{\frac{1}{2}} \Lambda^4 e^{-2\varphi}}{(1-e^{-\varphi})} + \frac{4\kappa^{\frac{3}{2}} \Lambda^4}{3} - \frac{40}{3} \kappa^{\frac{3}{2}} \Lambda^4 e^{-2\varphi} + \right. \\ \left. \frac{44\kappa^{\frac{3}{2}} \Lambda^4 e^{-\varphi}}{3} \right) \quad (37)$$

The spectral index and tensor-to-scalar ratio are modified accordingly to be

$$n_s = 1 - \frac{4e^{-2\varphi}}{\kappa(1-e^{-\varphi})^2} - \frac{4e^{-\varphi}}{\kappa(1-e^{-\varphi})^2} + \alpha \left(\frac{32}{\sqrt{\kappa}(1-e^{-\varphi})} - \frac{16e^{-\varphi}}{\sqrt{\kappa}(1-e^{-\varphi})} + \frac{8\varphi e^{-2\varphi}}{\sqrt{\kappa}(1-e^{-\varphi})^2} \right) + \\ \beta \left(-\frac{32}{3} \kappa^{\frac{1}{2}} \Lambda^4 + \frac{16}{3} \kappa^{\frac{1}{2}} \Lambda^4 e^{-2\varphi} + -8\kappa^{\frac{1}{2}} \Lambda^4 e^{-\varphi} + \frac{8}{3} \kappa^{\frac{1}{2}} \Lambda^4 e^{\varphi} + \frac{64\kappa^{\frac{1}{2}} \Lambda^4 e^{-3\varphi}}{3(1-e^{-\varphi})} - \right. \\ \left. -\frac{64}{3} \kappa^{\frac{3}{2}} \Lambda^4 e^{-2\varphi} + \frac{4}{3} \kappa^{\frac{3}{2}} \Lambda^4 + 8\kappa^{\frac{3}{2}} \Lambda^4 e^{-\varphi} - \frac{28\kappa^{\frac{3}{2}} \Lambda^4 e^{-\varphi}}{3} \right) \quad (38)$$

$$r = \frac{32e^{-2\varphi}}{\kappa(1-e^{-\varphi})^2} + \alpha \left(\frac{128e^{-\varphi}}{\kappa^{\frac{1}{2}}(1-e^{-\varphi})} - \frac{64\varphi e^{-2\varphi}}{\kappa^{\frac{1}{2}}(1-e^{-\varphi})^2} \right) + \beta \left(-\frac{256\kappa^{\frac{1}{2}} \Lambda^4 e^{-2\varphi}}{3(1-e^{-\varphi})} + \frac{128}{3} \kappa^{\frac{3}{2}} \Lambda^4 e^{-2\varphi} - \right. \\ \left. 128\kappa^{\frac{3}{2}} \Lambda^4 e^{-\varphi} \right) \quad (39)$$

In this way, the spectral index of scalar perturbations and the ratio of tensor perturbations to scalar ones are expressed. The spectral index and tensor-to-scalar ratio are calculated by (21) and their MCMC corner plot present in Figure 3. We have three free parameters α, β . The other internal parameters are expressed in terms of the free parameters. Table 3 shows the mean values of the coupling constants α and β for reconstruction with the Starobinsky potential.

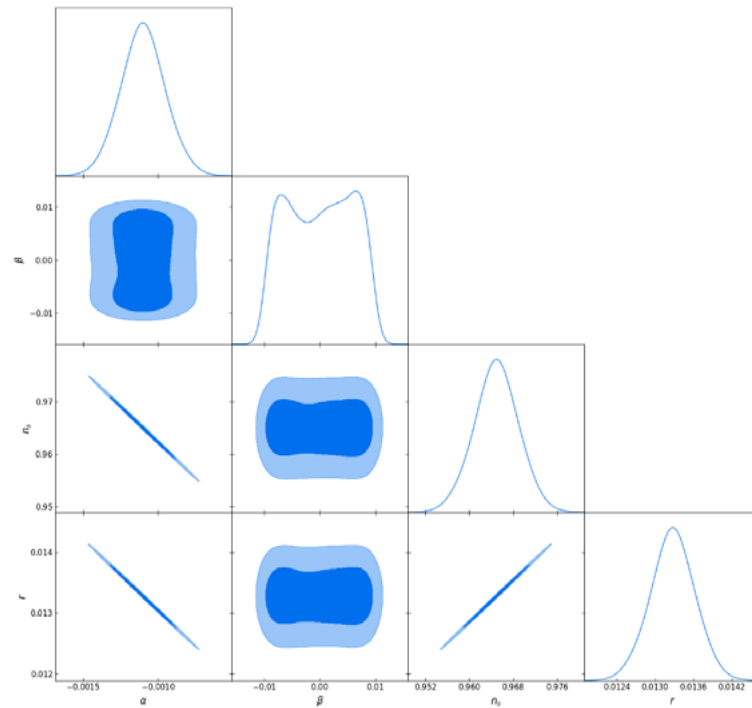


Figure 3. The spectral index and tensor-to-scalar ratio for the Starobinsky inflation potential (35). The range of α is $[-3.6 \pm 12.3] \times 10^{-5}$, β is "No limit". Marginalized joint 68% and 95% C.L. regions for α and β at n_s from Planck 2018 data [4] are shown in blue and light blue, respectively.

Table 3. Parameter values in the 1σ confidence interval for the Starobinsky inflation potential (35).

α	β	n_s	r
$(-3.6 \pm 12.3) \times 10^{-5}$	No limit	0.9650 ± 0.00392	0.00217 ± 0.00012

Conclusion

The article presents a study of a cosmological model with a scalar field containing a non-minimal coupling with a modified Gauss-Bonnet invariant $\beta f(G)f(\phi)$ and a trace of the energy-momentum tensor of the scalar field $\alpha f(\phi)T$ in the framework of a flat, homogeneous and isotropic Friedman-Robertson-Walker Universe. To study the inflationary properties of the model, the probabilities for the chaotic potential, the natural potential, and the Starobinsky potential are calculated. For the chaotic potential $\alpha^{bestfit} = -1.0983 \cdot 10^{-3}$, $\beta^{bestfit} = nonconstrained$ means that the Planck 2018 data impose restrictions on the non-minimal coupling of $\beta^{bestfit} = nonconstrained$ and the absence of restrictions on the non-minimal coupling of $\beta f(G)f(\phi)$. For the natural potential $\alpha^{bestfit} = -1.0983 \cdot 10^{-3}$, $\beta^{bestfit} = nonconstrained$, which corresponds to the same consequences as for the chaotic potential. For the Starobinsky potential, $\alpha^{bestfit} = -3.7510 \cdot 10^{-5}$, $\beta^{bestfit} = nonconstrained$, means that the Planck2018 data does not require modification of the equations of motion due to the introduction of a non-minimal coupling $\alpha f(\phi)T$ in the case of the Starobinsky potential, there are no restrictions on the non-minimal coupling $\beta f(G)f(\phi)$. Generalize classification of nonminimal coupling Einstein-Gauss-Bonnet with $\alpha f(\phi)T$ present in Table 4.

Table 4. Classification cosmological model

Potential	α / β constrained		Planck 2018 agriment
Chaotic	Yes/No	$3.6021 \cdot 10^{-2} \pm 1.2572 \cdot 10^{-2}$	No
Natural	Yes/No	$1.3270 \cdot 10^{-2} \pm 3.6207 \cdot 10^{-4}$	Yes
Starobinsky	No/No	$2.1717 \cdot 10^{-3} \pm 1.2794 \cdot 10^{-4}$	Yes

Acknowledgement, conflict of interests

This research was funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP26194585).

No conflict of interest.

The contribution of the authors

P.Yu. Tsyba conceptualization, methodology, software, validation, formal analysis, investigation, data curation, writing—original draft preparation, writing—review and editing, visualization, funding acquisition, funding acquisition.

Zh. Oktyabrkyzy methodology, software, validation, formal analysis, investigation, data curation, writing—original draft preparation.

U.A. Ualikhanova methodology, formal analysis, investigation, visualization.

M.V. Simakin formal analysis, investigation, data curation, visualization.

Conflict of Interest

The authors declare that they have no conflict of interest related to the publication of this article.

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**Энергия-импульс тензорының ізімен минималды байланысы
бар инфляцияға арналған ауырлық күшінің модификацияланған теориясының
модельдері скаляр өрісі және Гаусс-Бонн инварианты**

Аңдатпа. Мақалада модификацияланған Гаусс-Бонне инварианты және Фридман-Робертсон-Уоккер жазық, біртекті және изотропты ғаламның бөлігі ретінде скаляр өрісінің энергия-импульс тензорының ізімен минималды байланысы бар скаляр өрісі бар космологиялық модельдің инфляциялық қасиеттері зерттеледі. Модельдің инфляциялық қасиеттерін зерттеу үшін хаотикалық потенциал, табиғи потенциал және Старобинский потенциалы таңдалды. Баяу жылжудың жуықтауы шеңберінде және байланыс константаларын кіші деп санау (және) скалярлық спектрлік индексі мен скалярлық-тензорлық қатынас үшін жалпыланған өрнектер алынды. Скалярлық спектрлік индекс пен скалярлық-тензорлық қатынастың теориялық мәндерін есептеу және байқалғандармен салыстыру үшін космологиялық үшін қабық қолданылды. Собау cosmology есептеу Polychord сынағасымен бірге. Хаотикалық потенциал үшін бақыланатын Planck2018 деректеріне тек скалярлық спектрлік индекстің мәні сәйкес келетіні көрсетілген, ал скалярлық-тензорлық қатынастың мәні бақыланатын мәннен асып түседі. Старобинскийдің табиғи әлеуеті мен әлеуеті үшін

инфляциялық параметрлердің екі мәні де бақылауларға сәйкес келеді. жағдайы және потенциалдардың барлық түрлері үшін байланыс параметрі әр модельде маргиналданады, ал байланыс параметрі инфляциялық параметрлердің динамикасына әсер етпейді және шектеу қойылмайды.

Түйін сөздер: инфляция, Гаусс-Бонне инварианты, энергия-импульс тензоры, баяу домалауға жақындау, скалярлық спектрлік индекс, скалярлық-тензорлық қатынас, бақылау шектеулері.

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**Модели модифицированной теории гравитации для инфляции с неминимальной
связью со следом тензора энергии-импульса
скалярного поля и инвариантом Гаусса-Бонне**

Аннотация. В статье изучаются инфляционные свойства космологической модели со скалярным полем, содержащей неминимальную связь с модифицированным инвариантом Гаусса-Бонне и следом тензора энергии-импульса скалярного поля в рамках плоской, однородной и изотропной Вселенной Фридмана-Робертсона-Уокера. Для изучения инфляционных свойств модели были выбраны хаотический потенциал, природный потенциал и потенциал Старобинского. В рамках приближения медленного скатывания и при условии малости констант связи α и β получены обобщенные выражения для скалярного спектрального индекса n_s и скалярно-тензорного отношения r . В работе для вычисления теоретических значений скалярного спектрального индекса n_s и скалярно-тензорного отношения r , а также сравнения с наблюдаемыми была применена оболочка для космологических вычислений Cobaya cosmology совместно с сэмплером Polychord. Показано, что для хаотического потенциала согласуется с наблюдаемыми данными Planck2018 только значение скалярного спектрального индекса, а значение скалярно-тензорного отношения превышает наблюдаемое значение. Для природного потенциала и потенциала Старобинского оба значения инфляционных параметров согласуются с наблюдениями. Показано, что для случая $\alpha = \beta = 0$ и всех типов потенциалов параметр связи α маргинализируется в каждой модели, а параметр связи β не влияет на динамику инфляционных параметров и никак не ограничен.

Ключевые слова: инфляция, инвариант Гаусса-Бонне, тензор энергии-импульса, приближение медленного скатывания, скалярный спектральный индекс, скалярно-тензорное отношение, наблюдательные ограничения.

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