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Initial Conditions of Inflaton Field at the Quantum Bounce

Mohd Shahalam*

Department of Physics, Integral University, Lucknow, India (E-mail: mohdshahamu@gmail.com)

*(E-mail: *mohdshahamu@gmail.com)*

Abstract. In this paper, we explore the pre-inflationary evolution of the universe driven by a logarithmic potential in the context of Loop Quantum Cosmology. Our analysis focuses on identifying the physically admissible initial conditions for the inflaton field that result in a successful phase of slow-roll inflation. We also evaluate the corresponding number of e-folds and examine their consistency with current observational bounds. When the kinetic energy of the inflaton dominates at the initial stage, the cosmic evolution prior to reheating naturally separates into three successive phases: the bouncing phase, the transition phase, and the slow-roll inflationary phase. Throughout the bouncing phase, the dynamics of the scale factor are largely insensitive to both the chosen initial conditions and the detailed structure of the inflationary potential. In this regime, the evolution admits an explicit analytical solution, providing a clear and model-independent description of the background behavior before the onset of inflation. In this model, the quantum bounce is governed entirely by kinetic energy, since potential energy-dominated initial conditions cannot be realized over the full range of inflaton field.
Keywords: Big Bang Singularity, Quantum Bounce, Inflation, Loop Quantum Cosmology.

Introduction

The theory of cosmic inflation is a cornerstone of modern cosmology, providing a compelling explanation for several fundamental puzzles associated with the early universe. First proposed in the early 1980s by Alan Guth and others, inflation refers to a brief epoch of cosmic acceleration, nearly exponential expansion that occurred roughly between 10-36 and 10-32 seconds after the Big Bang. During this phase, the dynamics were driven by a high-energy scalar field known as the inflaton, whose potential energy temporarily dominated the total energy density of the universe, causing rapid expansion. Before the onset of inflation, regions that are now widely separated were causally connected, allowing them to achieve thermal equilibrium. The subsequent accelerated expansion stretched these regions far

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*the corresponding author

beyond each other's horizons, thereby explaining the remarkable isotropy of the cosmic microwave background (CMB). In this way, inflation successfully resolves the horizon and flatness problems of the standard cosmological model. Moreover, quantum fluctuations of the inflaton field provide the seeds for primordial density perturbations, which later evolved into the large-scale structure of the universe [1,2]. Over the years, a wide variety of inflationary models have been developed and confronted with observational data. In single-field inflation, the quadratic potential has been strongly disfavored relative to Starobinsky model, as indicated by the 2018 results of the Planck Collaboration [3]. In this paper, we numerically investigate the dynamics of the loop inflation using the corresponding background equations.

Despite its successes, inflation within the framework of classical General Relativity (GR) inevitably encounters the Big Bang singularity, where physical quantities such as curvature and energy density diverge [4,5]. Consequently, inflationary spacetimes remain geodesically incomplete in the past. An appealing resolution to this problem is provided by Loop Quantum Cosmology (LQC), in which the classical singularity is replaced by a non-singular quantum bounce [6-8]. LQC is derived from Loop Quantum Gravity (LQG), a background-independent approach to quantizing spacetime. In LQG, geometry is fundamentally discrete and described in terms of spin networks, implying that space itself has an underlying lattice-like structure at the Planck scale. When these principles are applied to homogeneous and isotropic cosmological settings, the resulting quantum corrections modify the classical Friedmann equations at high densities. As a consequence, instead of diverging to infinity, the energy density reaches a maximum finite value, leading to a quantum bounce that connects a contracting phase to the present expanding universe. This resolution of the initial singularity addresses one of the most profound shortcomings of classical cosmology. Furthermore, LQC makes distinctive predictions for the early-universe dynamics at high curvature scales, which may be tested through precise measurements of the CMB and primordial gravitational waves. Remarkably, it has been shown that the post-bounce evolution in LQC naturally gives rise to a phase of slow-roll inflation for a broad range of initial conditions [9-17]. To investigate the pre-inflationary dynamics and cosmological perturbations, two principal approaches are commonly employed: the dressed metric approach and the deformed algebra approach. However, when attention is restricted to the background evolution of the universe, both frameworks yield the same set of dynamical equations. Consequently, the results presented in this paper are applicable to either approach. A similar analysis can also be extended to other inflationary models. Nevertheless, the main conclusions derived here are expected to remain valid in other models as well, at least in cases where the early evolution of the universe is dominated by the kinetic energy of the inflaton field.

Loop Quantum Cosmology and the Quantum Bounce

In standard cosmology, the Big Bang is typically associated with a spacetime singularity—a state at which the energy density and curvature diverge, causing the classical equations of GR to lose their validity. Loop Quantum Cosmology introduces quantum geometric corrections to the Einstein field equations of General Relativity, thereby altering the high-energy behavior of the universe. As a consequence, the Friedmann equations are modified to incorporate these quantum effects [18,19].

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right) \tag{1}$$

The correction term $\rho\left(1 - \frac{\rho}{\rho_c}\right)$ encodes the quantum geometric effects that become dominant at high energy densities. This modification prevents the divergence of physical quantities by ensuring that the energy density never exceeds the critical value ρ_c [20,21]. As ρ approaches ρ_c, the expansion rate gradually decreases and eventually vanishes, resulting in a non-singular quantum bounce instead of a classical singularity. Consequently, the universe transitions smoothly from a contracting phase to an expanding one. The modified Friedmann equation in LQC therefore establishes a consistent connection between classical cosmological dynamics and the underlying quantum description of spacetime. The Klein-Gordon equation remains the same as in GR.

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0. \tag{2}$$

From Eq. (1), it follows that the Hubble parameter (H) vanishes when the energy density reaches the critical value $\rho=\rho_c$. This condition signals the occurrence of the quantum bounce. In the following, we investigate both the bouncing phase and the subsequent slow-roll inflation for the potential of the form given below.

The flatness of an inflationary potential is generally modified by radiative corrections. At one-loop order, these corrections typically appear in the form of a logarithmic term, $\ln(\phi/\mu)$, where μ denotes the renormalization scale. Even if one begins with an exactly flat potential, the inclusion of quantum effects leads to the emergence of a logarithmic potential. Investigating such potentials, therefore, provides a straightforward framework for understanding under what conditions quantum corrections spoil the flatness of the potential and the mechanism by which this occurs. Let us now consider the slow-roll analysis of loop inflation. The corresponding potential is given by

$$V(\phi) = V_0\left(1+\alpha \ln\frac{\phi}{m_{Pl}}\right) \tag{3}$$

where α is a dimensionless parameter that can take either positive or negative values. In this work, we shall use $\alpha=1$ and $V_0=1.12436*10^{-12}\, m_{Pl}^4$, see Appendix of Ref. [16]. At the bounce, we have

$$\begin{gathered} \rho=\rho_c \\ \frac{1}{2}\dot{\phi}^2(t_B)+V(\phi(t_B)) = \rho_c \\ \dot{a}(t_B) = 0 \end{gathered} \tag{4}$$

This implies that

$$\dot{\phi}(t_B) = \pm\sqrt{2\left(\rho_c - V\left(\phi(t_B)\right)\right)} \tag{5}$$

$$a(t_B) = 1$$

Here after, we shall read $\phi(t_B), \dot{\phi}(t_B)$ and $a(t_B)$ as $\phi_B, \dot{\phi}_B$ and a_B in the article. We introduce the key parameters that will be employed throughout the analysis.

(i) Equation of State: The equation of state (EoS) parameter for the scalar field is defined as

$$w(\phi) = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)} \simeq -1, \textit{in the slow-roll phase} \qquad (6)$$

To classify the nature of the initial conditions at the bounce, we evaluate $w(\phi)$ at $\phi=\phi_B$. Accordingly, we consider two types of initial conditions:

- **Kinetic Energy Dominated (KED):** $w(\phi_B)>0$
- **Potential Energy Dominated (PED):** $w(\phi_B)<0$

At the quantum bounce, the scale factor is fixed to $a_B=1$. However, the value of $w(\phi_B)$ distinguishes the two scenarios: it is positive for KED initial conditions and negative for PED initial conditions.

(ii) Slow-Roll Parameter: The Hubble slow-roll parameter $\boldsymbol{\epsilon_H}$ is defined as

$$\epsilon_H = -\frac{\dot{H}}{H^2} \ll 1, \textit{in the slow-roll phase} \qquad (7)$$

(iii) Number of e-Folds: The number of e-folds generated during inflation is given by

$$N_{inf} = \text{Log}\,(a_{end}/a_i) \qquad (8)$$

This quantity measures the total amount of accelerated expansion during inflation.

(iv) Analytical Scale Factor in the Bouncing Regime: In the vicinity of the bounce, the analytical form of the scale factor is given by

$$a(t) = a_B \left(1+\delta \frac{t^2}{t_{Pl}^2}\right)^{1/6} \qquad (9)$$

where $a_B=a(t_B)$, δ is a dimensionless parameter, and $t_{Pl,}$ denotes the Planck time. We numerically solve the modified Friedmann equation (1) and the Klein–Gordon equation (2) together with the chosen potential (3). At the bounce, we examine only positive inflaton velocity (PIV) $\dot{\phi}_B > 0$. However, one can also perform similar analysis for negative inflaton velocity (NIV). $\dot{\phi}_B < 0$The initial scalar field configurations are categorized into KED and PED cases. Let us analyze the case of PIV for both KED and PED initial conditions. The numerical evolution of the scale factor a(t), the EoS parameter $w(\phi)$, and the slow-roll parameter ϵ_H for various values of ϕ_B are illustrated in Fig. 1. First, we investigate the KED initial conditions of inflaton field. In the bouncing regime, the evolution of a(t) is largely independent of the specific choice of ϕ_B and closely follows the analytical solution (7). During the slow-roll phase, the scale factor exhibits exponential growth. The numerical evolution can be clearly divided into three stages:

1. **Bouncing phase:** $w(\phi)\approx+1$
2. **Transition phase:** $w(\phi)$ evolves from +1 to −1
3. **Slow-roll phase:** $w(\phi)\approx-1$

Furthermore, we compute the number of e-folds N_{inf} for different values of ϕ_B, and the results are summarized in Table 1. According to the 2018 results of the Planck Collaboration, a successful inflationary scenario requires at least 60 e-folds. This observational constraint places bounds on the allowed range of the initial field value ϕ_B. The values of the number of e-folds N_{inf} are summarized in Table 1. It is evident that N_{inf} increases with increasing initial field value ϕ_B. Similar trends have been reported in previous studies. We now turn to the PED initial conditions; in this case, the initial conditions at the bounce are governed solely by kinetic energy, since PED initial conditions do not arise at any stage of the bouncing phase. Similar conclusions were reported for the T-model in Ref. [16]. Hence, the PED initial conditions of inflaton field cannot be imposed across the entire range of ϕ_B.

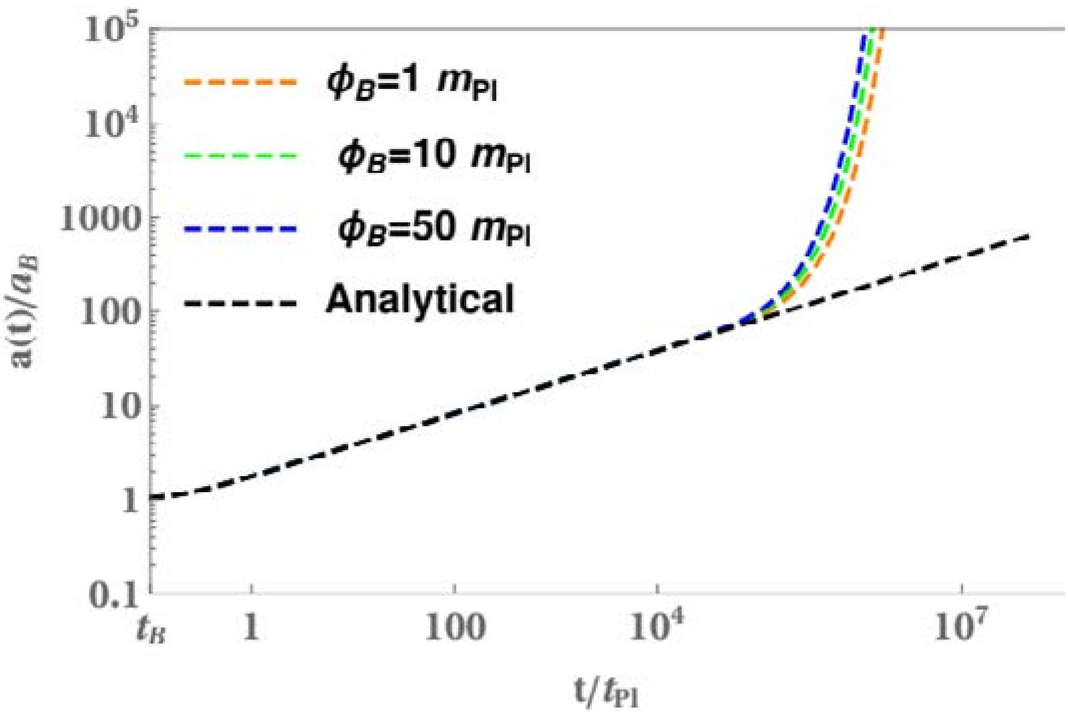

Figure 1. The figure illustrates the evolution of a(t) for different KED initial conditions at the bounce. The analytical solution for a(t) is also shown to enable a direct comparison with the numerical results. In this scenario, the evolution of a(t) exhibits universal behavior.

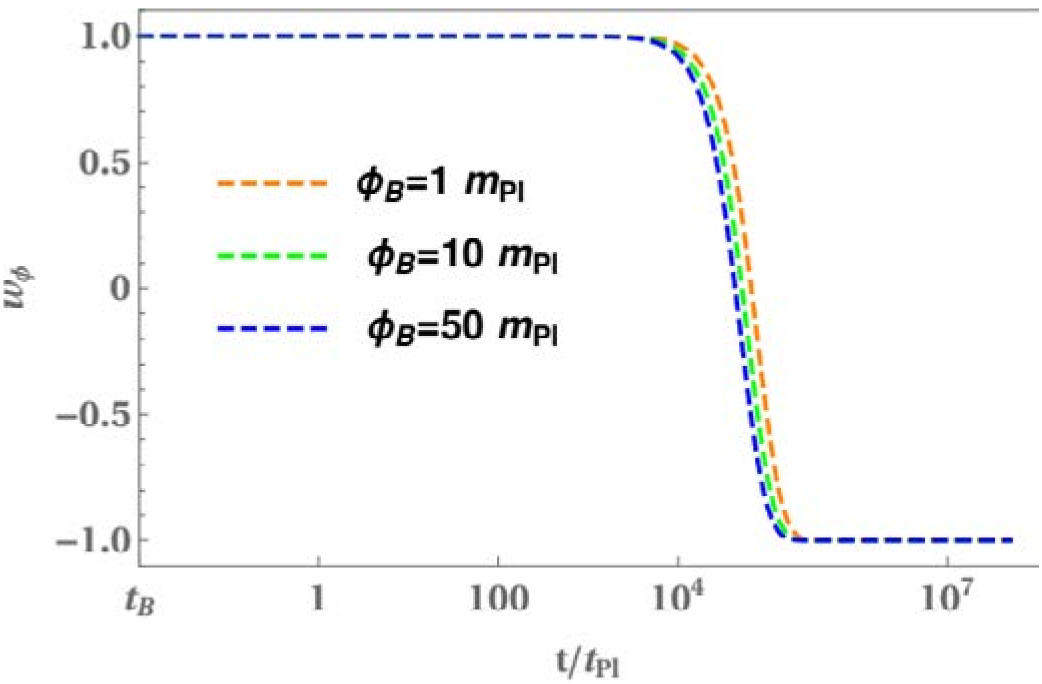

Figure 2. The figure exhibits the evolution of w(ϕ) for various KED initial conditions at the bounce. We observe that the evolution of the universe can be divided into three distinct phases:
the bouncing phase, the transition phase, and the slow-roll phase.

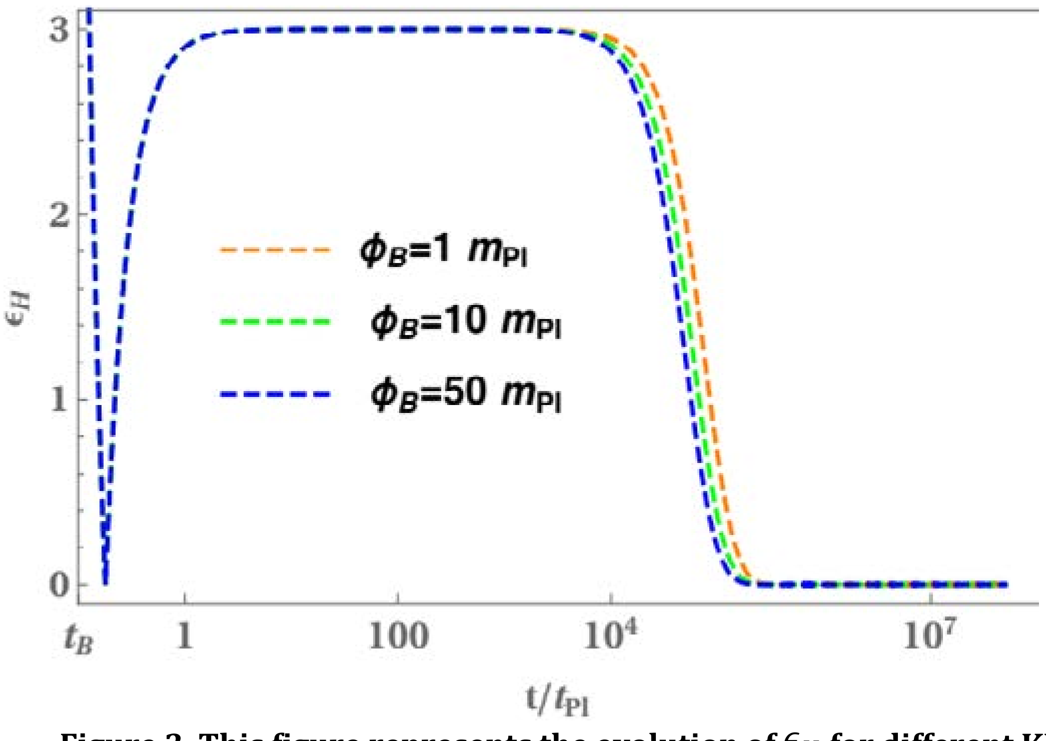

Figure 3. This figure represents the evolution of ϵ_H for different KED initial conditions at the bounce.

Table 1. Inflationary parameters for the potential (3) with $\phi_B > 0$.

ϕ_B/m_{Pl}	Inflation	t/t_{Pl}	ϵ_H	$w(\phi)$	N_{inf}
0.5	Start Slow-roll End	91045.7 376311 $1.0747*10^7$	0.92 0.00014 0.28	-1/3 -1 -1/3	45.55
1.0	Start Slow-roll End	82205.7 381749 $5.8796*10^7$	1.0 0.000018 0.33	-1/3 -1 -1/3	60.13
2.0	Start Slow-roll End	79532.6 386300 $1.0362*10^7$	1.0 0.00019 0.22	-1/3 -1 -1/3	62.23

Conclusions

In this work, we have investigated the pre-inflationary dynamics associated with the potential introduced in Eq. (3) within the framework of LQC. Our analysis shows that, for KED initial conditions of the inflaton field at the bounce, the cosmic evolution naturally divides into three well-defined stages: the bouncing phase, the transition phase, and the slow-roll inflationary phase. During the bouncing regime, the scale factor a(t) displays a universal behavior that remains largely insensitive to a broad range of initial values of ϕB, as well as to the detailed form of the inflationary potential. This universality is a distinctive feature of LQC when the kinetic energy dominates at the bounce. The numerical solutions for a(t) are found to be in excellent agreement with the analytical expression provided in Eq. (7), as demonstrated in Fig. 1. In this phase, the EoS parameter satisfies $w(\phi)\approx+1$, confirming that the dynamics are indeed governed by the kinetic term of the inflaton field. As the evolution proceeds, the universe enters a brief transition phase characterized by a rapid change in the EoS parameter from +1 to -1. This intermediate stage is considerably shorter in duration compared with both the bouncing and the subsequent slow-roll phases. Despite its short timescale, it plays a crucial role in connecting the kinetic energy-dominated bounce to the inflationary attractor solution. Following the transition, the universe moves into an accelerating regime. At the beginning of this stage, the slow-roll parameter is relatively large, reflecting the residual effects of the preceding dynamics. However, it quickly decreases and approaches a value close to zero, indicating that the system is settling into the slow-roll inflationary phase. This behavior is clearly illustrated in Figures 2 and 3. The total number of e-folds Ninf generated during the slow-roll period has been computed for various initial conditions and is summarized in Table 1. The results confirm that a sufficient number of e-folds can be achieved for a suitable range of ϕB, consistent with observational requirements. We also examined the scenario corresponding to PED initial conditions. Interestingly, even in this case, the dynamics at the bounce are effectively controlled by the kinetic energy of the inflaton. In other words, PED conditions do not genuinely arise during the bouncing phase, as the kinetic term inevitably dominates the energy density at that stage. Similar findings were reported for the T-model in Ref. [16]. Consequently, PED initial conditions for the inflaton field cannot be consistently implemented across the entire range of ϕB, reinforcing the robustness of the KED bounce in this framework.

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Conflict of interests

There is no conflict of interests.

The contribution of the author.

Mohd Shahalam – The entire paper is executed by a single author.

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Мохд Шаhалам

Физика кафедрасы, Интеграл университет, Лакхнау, Үндістан

(E-mail: mohdshahamu@gmail.com)

Кванттық секіру кезіндегі инфляциялық өрістің бастапқы шарттары

Аңдатпа. Бұл мақалада циклдік кванттық космология контексінде логарифмдік потенциалмен басқарылатын әлемнің инфляцияға дейінгі эволюциясын зерттейміз. Біздің талдауымыз баяу домалау инфляциясының сәтті фазасына әкелетін үрлеу өрісі үшін физикалық тұрғыдан рұқсат етілген бастапқы жағдайларды анықтауға бағытталған. Сондай-ақ тиісті электронды қатпарлар санын бағалаймыз және олардың ағымдағы бақылау шекараларымен сәйкестігін тексереміз. Инфляцияның кинетикалық энергиясы бастапқы кезеңде басым болған кезде, қайта қыздыруға дейінгі ғарыштық эволюция үш кезеңнен тұрады: секіру фазасы, өтпелі фаза және баяу домалау инфляция фазасы. Секіру фазасы бойы масштаб факторының динамикасы таңдалған бастапқы жағдайларға да, инфляциялық потенциалдың егжей-тегжейлі құрылымына да сезімтал емес. Бұл режимде эволюция инфляция басталғанға дейінгі фондық динамика анық және модельге тәуелсіз сипаттамасын ұсына отырып, айқын аналитикалық шешімге ие. Бұл модельде кванттық секіру толығымен кинетикалық энергиямен басқарылады, себебі потенциалдық энергия басым бастапқы жағдайлар инфляциялық өрістің толық диапазонында жүзеге аспайды.

Түйін сөздер: үлкен жарылыс сингулярлығы, кванттық секіру, инфляция, циклдік кванттық космология

Мохд Шахалам
Кафедра физики, Интегральный университет, Лакхнау, Индия
(E-mail: mohdshahamu@gmail.com)

Начальные условия поля инфлатона при квантовом отскоке

Аннотация. В данной работе мы исследуем прединфляционную эволюцию Вселенной, обусловленную логарифмическим потенциалом, в контексте петлевой квантовой космологии. Наш анализ сосредоточен на определении физически допустимых начальных условий для поля инфлатона, которые приводят к успешной фазе медленной инфляции. Мы также оцениваем соответствующее число e-складок и изучаем их согласованность с текущими наблюдательными ограничениями. Когда кинетическая энергия инфлатона доминирует на начальном этапе, космическая эволюция до перенагрева естественным образом разделяется на три последовательные фазы: фазу отскока, переходную фазу и фазу медленной инфляции. На протяжении всей фазы отскока динамика масштабного фактора в значительной степени нечувствительна как к выбранным начальным условиям, так и к детальной структуре инфляционного потенциала. В этом режиме эволюция допускает явное аналитическое решение, обеспечивающее четкое и независимое от модели описание фонового поведения до начала инфляции. В этой модели квантовый отскок полностью определяется кинетической энергией, поскольку начальные условия, в которых доминирует потенциальная энергия, не могут быть реализованы во всем диапазоне поля инфлатона.

Ключевые слова: сингулярность Большого взрыва, квантовый отскок, инфляция, петлевая квантовая космология

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Information about the author:

Mohd Shahalam – the corresponding author, Ph.D., Professor, Department of Physics, Integral University, Lucknow 226026, India

Мохд Шахалам – хат-хабар авторы, Ph.D., Физика кафедрасының профессоры, Интеграл университеті, 226026, Лакхнау, Үндістан.

Мохд Шахалам – автор для корреспонденции, PhD, профессор кафедры физики Интегрального университета, Лакхнау 226026, Индия.