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Slow-roll inflation in the power-law scalar model

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Annotation. In this work, we examine an inflationary model driven by a scalar field that evolves according to a power–law dependence on cosmological time. This assumption allows the Einstein–Klein–Gordon equations to be solved analytically within a flat FLRW metric and makes it possible to obtain closed-form expressions for the key dynamical quantities of the early Universe. Based on the chosen scalar field profile, the Hubble function and the scale factor are computed, enabling a detailed analysis of the emergence of accelerated expansion. The potential of the field and its derivatives are reconstructed directly from the equation of motion, which in turn allows us to derive analytical formulas for the Hubble and potential slow-roll parameters. The time evolution of these parameters is analyzed to determine the regime of validity of the slow-roll approximation and to identify the natural endpoint of the inflationary phase. The obtained results demonstrate that a power-law configuration of the scalar field can sustain a prolonged stage of inflation and accurately reproduce the major features of slow-roll dynamics within the framework of General Relativity.
Keywords: Inflation field, slow-roll parameter, generalized gravity, Hubble parameter, scale factor, Klein-Gordon equation.

Introduction

The inflationary framework has become a cornerstone of modern cosmology, as it provides a unified mechanism for explaining several essential properties of the observable universe, including its large-scale homogeneity, flat spatial geometry, and the absence of relics predicted by pre-inflationary models. In this scenario, the universe undergoes a brief period of accelerated expansion in its earliest stages, during which quantum fluctuations are stretched to cosmological scales and subsequently evolve into the density perturbations detected in the cosmic microwave background.

Within the context of Einstein's General Relativity, such rapid expansion can be generated by a single scalar field,the inflator-whose potential energy dominates the total energy content of the universe. When the inflation evolves sufficiently slowly so that its potential energy

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remains the primary contribution, the dynamical equations simplify to the well-known slow-roll approximation. In this regime, the inflationary behavior is described by two small, dimensionless parameters:

$$\epsilon = -\frac{\dot{H}}{H^2}, \qquad \eta = \frac{\ddot{\varphi}}{H\dot{\varphi}}, \tag{1}$$

which measure the deviation from an exact de Sitter expansion and quantify the rate at which the scalar field descends along with its potential.

Different choices of the scalar potential *V(φ)* – including monomial, exponential, and power-law forms – lead to distinct predictions for inflationary observables. These theoretical outcomes can be evaluated through parameters such as the scalar spectral index n_s and the tensor-to-scalar ratio *r,* both of which are tightly constrained by recent observations from the *Planck* and *WMAP* missions.

The aim of this work is to investigate the dynamics of the slow-roll phase within General Relativity, derive analytical relationships among the principal inflationary parameters, and compare the resulting predictions with current observational bounds. Special emphasis is placed on understanding how the specific shape of the scalar potential influences the duration of inflation and the amplitude of primordial perturbations, thereby providing deeper insight into the early evolution of the universe.

Theoretical Framework

The starting point is the Einstein–Hilbert action with a minimally coupled scalar field,

$$S = \int \left(\frac{R}{2k} + L_m\right) \sqrt{-g} d^4x \tag{2}$$

where the Lagrangian is introduced as

$$L_m = \frac{1}{2}\dot{\varphi}^2 - V(\varphi). \tag{3}$$

The background geometry is described by the spatially flat FLRW line element,

$$ds^2 = -dt^2 + a^2(t)\,(dx^2 + dy^2 + dz^2), \tag{4}$$

which reduces the gravitational action to the effective point–like Lagrangian.

$$L = -\frac{3}{\kappa}\, a\dot{a}^2 + \frac{1}{2}\, a^3\dot{\varphi}^2 - a^3 V(\varphi). \tag{5}$$

From this form, the energy density and pressure of the scalar field follow directly:

$$\rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + V(\varphi), \qquad p_\varphi = \frac{1}{2}\dot{\varphi}^2 - V(\varphi). \tag{6}$$

The evolution of the background spacetime is governed by the Friedmann equations,

$$3H^2 = \rho_\varphi, \tag{7}$$

$$2\dot{H} + 3H^2 = -p_\varphi, \tag{8}$$

while the scalar field satisfies the Klein-Gordon equation,

$$\ddot{\varphi} + 3H\dot{\varphi} + V_{,\varphi} = 0. \tag{9}$$

Combining (5) and (6) yields the useful identity.

$$\dot{H} = -\frac{1}{2}\dot{\varphi}^2. \tag{10}$$

We assume that we have a scalar field value,

$$\varphi = \varphi_0 t^\lambda, \qquad \dot{\varphi} = \varphi_0 \lambda t^{\lambda-1}, \qquad \ddot{\varphi} = \varphi_0 \lambda(\lambda-1)t^{\lambda-2}. \tag{11}$$

Here φ_0 and λ are constants with $\varphi_0>0$ and $\lambda<0$. The power-law time dependence of the scalar field is adopted as a technically convenient and physically reasonable ansatz in inflationary cosmology. This choice makes it possible to integrate the Einstein-Klein-Gordon system in closed form and to track explicitly the temporal behavior of the Hubble parameter, the scale factor, and the slow-roll characteristics. Moreover, taking the exponent λ implies that the scalar field gradually decreases with time during the inflationary era, which matches the standard picture of a rolling inflation field and naturally leads to the end of inflation.

Substituting this ansatz into (9) gives

$$\dot{H} = -\frac{1}{2}\,\varphi_0^2\lambda^2\, t^{\,2\lambda-2}. \tag{12}$$

Integrating with respect to time leads to the Hubble expansion rate,

$$H = H_0 - \frac{\varphi_0^2\lambda^2}{2(2\lambda-1)}\, t^{\,2\lambda-1}, \tag{13}$$

where H_0 is integration constant. Using the relation $\dot{a}/a=H(t)$, the scale factor takes the form.

$$a = a_0 exp\left(H_0 t - \frac{\varphi_0^2\lambda^2}{4\lambda(2\lambda-1)}\, t^{\,2\lambda}\right). \tag{14}$$

where a_0 is a constant of integration. The evolution of the scale factor a(t) with respect to cosmic time t is presented in Figure 1.

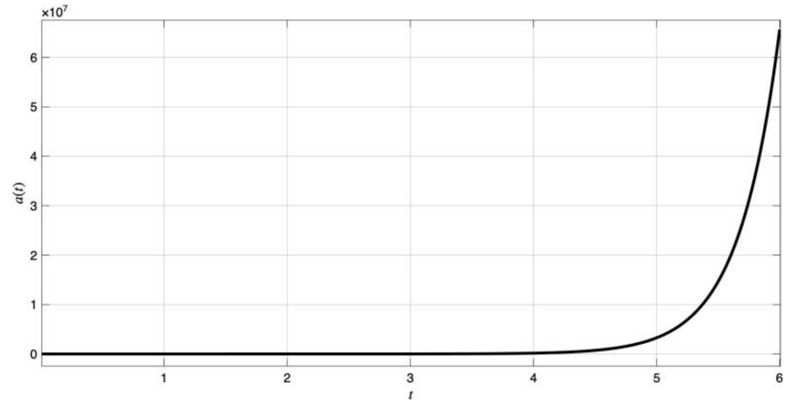

Figure 1. Behavior of the scale factor *a* as *a* function of cosmic time *t*. The monotonic growth of *a* reflects the accelerated expansion of the Universe during the slow-roll inflationary phase

Introducing the explicit formulas for the field derivatives leads to

$$V_{,\varphi} = -\varphi_0\lambda(\lambda-1)t^{\lambda-2} - 3H(t)\,\varphi_0\lambda\, t^{\lambda-1}, \tag{15}$$

and after replacing the equation (13) by its analytic expression,

$$V_{,\varphi} = -\varphi_0\lambda(\lambda-1)t^{\lambda-2} - 3H_0\varphi_0\lambda t^{\lambda-1} + \frac{3\varphi_0^3\lambda^3}{2(2\lambda-1)}\, t^{3\lambda-2}. \tag{16}$$

From the Klein-Gordon equation (15) by using the chain rule,

$$\frac{dV}{dt} = \frac{dV}{d\varphi}\frac{d\varphi}{dt} = V_{,\varphi}\,\dot{\varphi}, \tag{17}$$

Using equations (15), (16) we can derive V by integrating.

$$V = -\frac{1}{2}\,\varphi_0^2\lambda^2\, t^{\,2\lambda-2} - \frac{3H_0\,\varphi_0^2\lambda^2}{2\lambda-1}\, t^{\,2\lambda-1} + V_0. \tag{18}$$

To characterize the inflationary regime, the Hubble slow-roll parameters are introduced. The first is defined by.

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{\frac{1}{2}\varphi_0^2\lambda^2 t^{\,2\lambda-2}}{\left(H_0 - \frac{\varphi_0^2\lambda^2}{2(2\lambda-1)}\, t^{\,2\lambda-1}\right)^2}, \tag{19}$$

Figure 2 illustrates the temporal evolution of the slow-roll parameter ϵ.

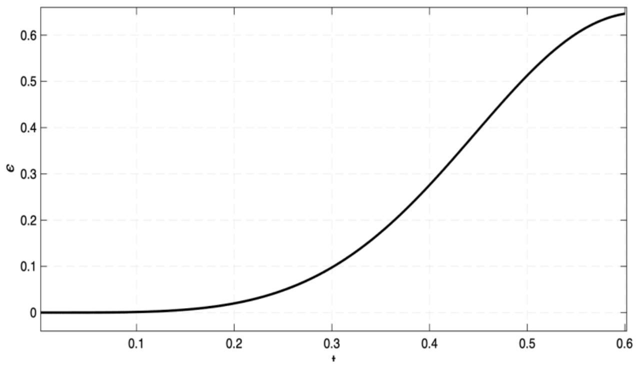

Figure 2. Evolution of the Hubble slow–roll parameter ϵ as *a* function of cosmic time *t*. The parameter remains much smaller than unity for most of the evolution, indicating a prolonged quasi–de Sitter stage, and gradually increases towards $\epsilon \simeq 1$, signalling the end of inflation

Therefore, we can define the first slow-roll parameter, denoted by ϵ, as while the second parameter, associated with the acceleration of the scalar field, becomes.

$$\eta = -\frac{\ddot{\varphi}}{H\dot{\varphi}} = -\frac{\lambda-1}{t\left(H_0-\frac{\varphi_0^2\lambda^2}{2(2\lambda-1)}\,t^{\,2\lambda-1}\right)}. \tag{20}$$

If we take the derivative with respect to cosmic time, we can define the second slow-roll parameter, denoted by η, which guarantees the slow variation of ϵ in time,

$$\dot{\epsilon} = \frac{2\dot{H}^2}{H^3} - \frac{\ddot{H}}{H^2} = \frac{1}{2}\,\varphi_0^4\lambda^4 t^{4\lambda-4}\left(H_0 - \frac{\varphi_0^2\lambda^2}{2(2\lambda-1)}t^{2\lambda-1}\right)^{-3} + \frac{\varphi_0^2\lambda^2(\lambda-1)t^{2\lambda-3}}{\left(H_0-\frac{\varphi_0^2\lambda^2}{2(2\lambda-1)}t^{2\lambda-1}\right)^2}. \tag{21}$$

$$\epsilon_V = \frac{1}{2\kappa}\left(\frac{-\varphi_0\lambda(\lambda-1)t^{\lambda-2}-3H_0\varphi_0\lambda t^{\lambda-1}+\frac{3\varphi_0^3\lambda^3}{2(2\lambda-1)}\,t^{3\lambda-2}}{-\frac{1}{2}\,\varphi_0^2\lambda^2t^{2\lambda-2}-\frac{3H_0\varphi_0^2\lambda^2}{2\lambda-1}t^{2\lambda-1}+V_0}\right)^2. \tag{22}$$

The behavior of the potential slow-roll parameter ϵ_V as a function of cosmic time *t* is presented in Figure 3.

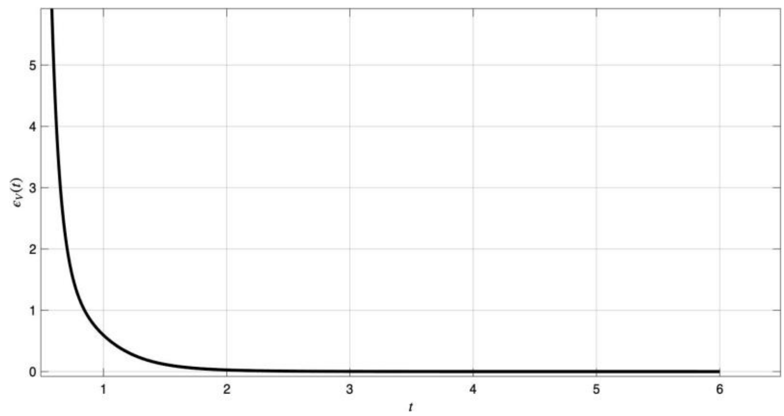

Figure 3. Behavior of the potential slow-roll parameter ϵ_V as *a* function of cosmic time *t*. Initially ϵ_V is small, confirming that the dynamics are potential-dominated, while its subsequent growth marks the breakdown of the slow–roll approximation and the end of the inflationary era

Here, figure 2 and figure 3 describe different slow-roll characteristics of the same inflationary dynamics. Figure 2 represents the Hubble slow-roll parameter ϵ which is sensitive to the full background evolution, including the kinetic term of the scalar field. Figure 3 shows the potential slow-roll parameter ϵ_V, which depends only on the form of the reconstructed potential. During the early stage, both parameters satisfy ϵ,ϵ_V indicating accelerated expansion. At later times both parameters increase and approach unity, which signals the breakdown of the slow-roll regime and confirms that the Universe exits inflation in the considered model.

Similarly to the case of ϵ_V, we can define a parameter η_V that depends only on the potential. Using (20) and the Friedmann equations, we have

$$\eta_V = \eta + \epsilon = \frac{1}{\kappa}\left(\frac{V_{,\varphi\varphi}}{V}\right), \tag{23}$$

where $V_{,\varphi\varphi} = \frac{d^2V}{d\varphi^2}$. To compute the potential slow-roll parameter η_V, we differentiate then we obtain.

$$V_{,\varphi\varphi} = -(\lambda-1)\left(\frac{\lambda-2}{t^2} + \frac{3H_0}{t}\right) + \frac{3\varphi_0^2\lambda^2(3\lambda-2)}{2(2\lambda-1)}\ t^{2\lambda-2}. \tag{24}$$

Finally, the potential slow-roll parameter is.

$$\eta_V = \frac{1}{\kappa}\ \frac{-(\lambda-1)\left(\frac{\lambda-2}{t^2}+\frac{3H_0}{t}\right)+\frac{3\varphi_0^2\lambda^2(3\lambda-2)}{2(2\lambda-1)}t^{2\lambda-2}}{-\frac{1}{2}\varphi_0^2\lambda^2\ t^{2\lambda-2}-\frac{3H_0\varphi_0^2\lambda^2}{2\lambda-1}\ t^{2\lambda-1}+V_0}. \tag{25}$$

The dependence of the parameter η_V on cosmic time t is shown in Figure 4.

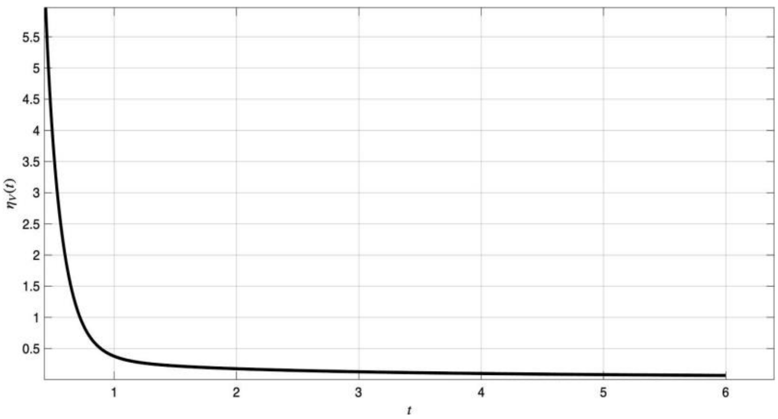

Figure 4. Behavior of the potential slow-roll parameter η_V as a function of cosmic time *t*. The evolution of η_V characterizes the curvature of the inflation potential and shows when the scalar field starts to deviate significantly from the slow-roll regime

Conclusion

In this work, we examined the inflationary dynamics generated by a power-law scalar field within the framework of General Relativity. By obtaining explicit analytical expressions for the Hubble parameter, the scale factor, and the slow-roll functions, we achieved a clear and fully transparent description of the background evolution during inflation. The slow-roll parameters ϵ and η remain sufficiently small throughout most of the inflationary epoch, confirming that the model naturally supports a prolonged phase of accelerated expansion. As cosmic time increases, both parameters gradually rise, signalling the eventual breakdown of the slow-roll regime. Taken together, these results demonstrate that a simple power-law dependence of the scalar field can reproduce the fundamental features of inflation and provides a coherent theoretical framework for describing the early Universe.

The contribution of the authors
Razina O.V. – formulation of the research tasks and overall scientific supervision.
Ratbay A. – preparation of the manuscript, analytical calculations, collection of material.
Ismailova A. – construction of graphs and computational support.

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Дәрежелік скаляр өрістік модельдегі баяу сырғу инфляциясы

Аннотация. Бұл жұмыста космологиялық уақытқа қатысты дәрежелік заң бойынша өзгеретін скаляр өрісі тудырған инфляциялық модель қарастырылады. Осындай болжам плоский FLRW метрикасында Эйнштейн–Клейн–Гордон теңдеулерін аналитикалық шешуге және ерте Ғаламның негізгі динамикалық шамаларының тұйық түрдегі өрнектерін алуға мүмкіндік береді. Берілген скаляр өріс профиліне сүйене отырып, Хаббл функциясы мен масштабтық фактор есептеледі, бұл жеделдетілген кеңеюдің қалыптасуын бақылауға жол ашады. Өрістің потенциалы мен оның туындылары қозғалыс теңдеуінен тікелей қалпына келтіріліп, Хабблдық және потенциалдық баяу сырғу параметрлерінің аналитикалық түрін алуға мүмкіндік береді. Аталған параметрлердің уақыттық эволюциясы баяу сырғу жуықтауының қолданылу аймағын және инфляциялық кезеңнің табиғи аяқталу уақытын анықтау үшін талданады. Алынған нәтижелер дәрежелік формадағы скаляр өрісінің конфигурациясы инфляцияның тұрақты кезеңін қамтамасыз етіп, Жалпы салыстырмалылық шеңберінде баяу сырғудың негізгі ерекше-ліктерін дәл сипаттай алатынын көрсетеді.

Түйінді сөздер: Инфляция, баяу сырғу параметрлері, жалпыланған гравитация, Хаббл параметрі, масштабтық фактор, Клейн-Гордон теңдеуі.

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Инфляция в режиме медленного скатывания в степенной модели скалярного поля

Аннотация. В работе рассматривается инфляционная модель, порождённая скалярным полем, эволюция которого задана степенной зависимостью от космического времени. Принятое предположение позволяет аналитически решить уравнения Эйнштейна–Клейна–Гордона в плоской FLRW–метрике и получить замкнутые выражения для динамических величин ранней Вселенной. На основе заданного вида поля вычисляются функция Хаббла и масштабный фактор, что даёт возможность проследить формирование ускоренного расширения. Потенциал скалярного поля и его производные восстанавливаются непосредственно из уравнения движения, что позволяет получить аналитические формы параметров медленного скатывания как хаббловского, так и потенциального типа. Поведение этих параметров анализируется для определения области действительности приближения медленного скатывания и момента выхода из инфляционного режима. Полученные результаты показывают, что простая степенная форма скалярного поля способна обеспечить устойчивую инфляционную стадию и воспроизвести характерные признаки медленного скатывания в рамках общей теории относительности.

Ключевые слова: Инфляция, параметры медленного скатывания, обобщённая гравитация, параметр Хаббла, масштабный фактор, уравнение Клейн-Гордона.

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