Л.Н. Гумилев атындағы Еуразия ұлттық университетінің ХАБАРШЫСЫ.
ISSN: 2616-6836. eISSN: 2663-1296

ФИЗИКА СЕРИЯСЫ/ PHYSICS SERIES / СЕРИЯ ФИЗИКА

IRSTI 29.05.23
https://doi.org/10.32523/2616-6836-2025-153-4-29-40
Article type: article

Extending the Covariant Confined Quark Model to Describe Radial Excitations of Heavy Quarkonia

Z.Z. Tyulemissov*, **A.E. Tyulemissova**

The Joint Institute for Nuclear Research, Dubna, Russia

*(E-mail: *zhomart.tyulemissov@gmail.com, akmaral.tyulemissova@gmail.com)*

Abstract. We present an extension of the Covariant Confined Quark Model (CCQM) that enables a consistent description of radial excitations of heavy quarkonia. The conventional CCQM formulation, based on static constituent quark masses, faces a limitation in describing states whose squared mass exceeds the sum of the squared constituent masses. To overcome this, we introduce a running constituent quark mass in the quark loop, allowing an accurate representation of excited hadronic decays. The binding energy of an excited state is assumed equal to that of the corresponding ground state, leading to a simple mass relation between quarkonia and their constituents. Model parameters are determined from experimental data on leptonic decays of charmonia and bottomonia, showing excellent agreement with observations. Furthermore, an orthogonality condition for radial excitations is implemented through modified vertex functions, ensuring that distinct excited states do not mix. This refinement leads to improved internal consistency and predictive power of the CCQM for excited hadronic systems.

Keywords: the covariant confined quark model, radial excitations, running constituent quark mass, quarkonium, charmonium, bottomonium, leptonic decay.

Introduction

The Covariant Confined Quark Model (CCQM) has proven to be a powerful and flexible framework for investigating a wide range of processes involving mesons and baryons. Within this model, hadrons are represented as bound states of constituent quarks, with confinement implemented through an infrared cutoff in the quark loop diagrams. The CCQM has successfully described numerous hadronic decay modes – strong [1-5], electromagnetic [6], semileptonic [7-9], and nonleptonic [10-12] – providing consistent predictions in good agreement with experimental data.

However, the conventional CCQM formulation employs static constituent quark masses, which restricts its applicability to ground states and low-lying hadrons. In particular, it fails

Received 20.10.2025. Revised 14.11.2025. Accepted 14.11.2025. Available online 25.12.2025

*the corresponding author

to describe the decays of highly excited mesonic states, where the squared mass of the hadron may exceed the sum of the squared constituent masses. This limitation prevents the model from capturing the full quark–antiquark dynamics in radial excitations of heavy quarkonia such as charmonium and bottomonium.

To overcome this problem, we propose an extension of the CCQM by introducing a running constituent quark mass in the internal quark loop. This modification preserves the basic principles of confinement and covariance while allowing the model to remain applicable across the full quarkonium spectrum. The running mass is determined by assuming that the binding energy of excited states is equivalent to that of the ground state, which provides a simple and physically transparent relation between quark and meson masses.

In addition, we formulate an orthogonality condition for radial excitations to ensure that different excited states do not mix within the model. This is achieved through a systematic construction of vertex functions that satisfy orthogonality constraints, leading to a self-consistent and hierarchically organized spectrum of quarkonium states.

The aim of this work is therefore twofold: to generalize the CCQM framework to accommodate radial excitations of heavy quarkonia through a running quark mass mechanism, and to implement the orthogonality condition for excited-state vertex functions, thereby improving the model's internal consistency and predictive capability.

The results presented here demonstrate that the extended CCQM reproduces the experimental data for leptonic decays of charmonia and bottomonia with high precision and offers a reliable tool for future studies of excited hadronic systems.

The Literature review

The study of heavy quarkonia has been approached using a variety of theoretical frameworks, each aiming to describe the mass spectra, decay constants, and other properties of these bound states. [13] employed a contact interaction model within the Dyson-Schwinger and Bethe-Salpeter equations, successfully reproducing masses, decay constants, and charge radii of charmonium states such as $\eta_c(1S), J/\psi(1S), \chi_{c0}(1P)$, *and* $\chi_{c1}(1P)$ in agreement with experimental data. Their approach emphasizes the role of symmetry-preserving interactions in modeling quark–antiquark dynamics.

[14] used a nonrelativistic potential model based on the Cornell potential, solving the Schrödinger equation numerically to calculate mass spectra and decay properties of heavy quarkonia. This approach demonstrates that simple potential models, calibrated with quark masses and coupling strengths, can provide reliable predictions compatible with experimental observations.

[15] extended the study of quarkonia using a light-front relativistic approach with holographic techniques, including one-gluon exchange and running couplings. Their framework allows for a detailed description of the mass spectrum, decay constants, and spatial distributions, producing results that reasonably match experimental data.

On the experimental side, the ALICE Collaboration [16] provided measurements of inclusive production cross-sections of J/ψ, $\Psi(2S)$, $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ in proton-proton collisions at

$\sqrt{s}$ = 8 TeV, giving essential input for testing theoretical models of quarkonium formation and decay dynamics.

While these studies offer important insights, many of them are limited in addressing the dynamics of radially excited states consistently across different quarkonia. In particular, models with fixed constituent quark masses face challenges in describing states whose squared mass exceeds the sum of squared constituent masses. Our work addresses this limitation by extending the Covariant Confined Quark Model (CCQM) with a running constituent quark mass mechanism and implementing orthogonal vertex functions, providing a general and internally consistent framework applicable to a broad range of heavy quarkonia and their excitations.

The Covariant Confined Quark Model

We start from represent basic principles and terms of the CCQM. The interaction Lagrangian is main object of the model. It describes essential properties and an interaction of constituent quark with hadron. General form of the Lagrangian has following form

$$L_{int}(x)=g\, H(x)J(x) + h.c., \qquad (1)$$

where g is a coupling constant corresponds to interaction of hadronic field H(x) with interpolating quark current J(x). In the case of quarkonium when meson consist from $\bar{q}q$ pair Hermitian conjugate part coincides with initial. The interpolating quark are presented as

$$J(x)=\iint dx_1\, dx_2\, F(x; x_1, x_2)\bar{q}(x_1)\Gamma q(x_2), \qquad (2)$$

here, $F(x; x_1, x_2)$ is a vertex function which defines the nonlocality of our model and Γ is γ-matrix of Dirac and can be chosen from quantum numbers J^{PC} of quarkonium. In coordinate space vertex function has following form

$$F(x; x_1, x_2) = \delta(x-w_1 x_1-w_2 x_2)\Phi[(x\_1-x_2)^2], \qquad (3)$$

δ-function is introduced for conservation of the translation invariance, second term $\Phi[(x_1-x_2)^2]$ is a function which is in the momentum space chosen to has exponent form to avoid any of ultraviolet divergence.

On first step of calculation, one has to expand the S-matrix in the second order of the interaction Lagrangian to obtain coupling constant. To proper computation of the coupling constant and avoid double counting of freedom degree of quarks one should use the so-called compositeness condition, which is expressed in terms of the derivative of the meson mass operator:

$$Z_M = 1 - g^2 \frac{d}{dp^2}\Pi(p^2) = 0 \qquad (4)$$

This condition was proposed by Salam and Weinber and means that quark degree of freedom excepts from physical state. In other words, quarks cannot be found in isolation. The relation corresponds to mass operator in the case of quarkonium state has the following form

$$\Pi(p^2) = \frac{1}{3}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) N_c \int \frac{d^4k}{(2\pi)^4 i} \widetilde{\Phi}(-k^2)\, \mathrm{tr}[\gamma^\mu S(k+w_1 p)\gamma^\nu S(k+w_2 p)], \quad (5)$$

where $g_{\mu\nu}$ is metric tensor, $N_c=3$ is number of quark colors, k is loop momenta, $w_1=w_2=1/2$, $S(k) = \frac{1}{m-\hat{k}}$ is a free propagator of a constituent quark with mass m.

The CCQM has six previously fixed parameters for ground states of hadrons, such as a constituent mass of quarks and cutoff parameter, which helps to avoid any of infrared divergence. The value of these parameters presented in Table 1.

Table 1. Model parameters: quark masses and cutoff parameter λ (in GeV units).

m_u/m_d	m_s	m_c	m_b	λ
0.241	0.428	1.67	5.04	0.181

There is also a size parameter of hadron, whose variation enables a proper reproduction of the observables. In the case of meson we take leptonic decay constant and attempting to fit our value to experimental or lattice QCD data. We then use this quantity to predict other properties. Table 2 represents masses and leptonic decay constants for radial excitations.

Table 2. Masses of charmonia and bottomnia from PDG and leptonic decay constants (in GeV units).

J/ψ	$\Psi(2S)$	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	$f_{(J/\psi)}$	$f_{\Upsilon(1S)}$
3.096900(6)	3.68610(6)	9.46030(26)	10.02326(31)	10.3552(5)	0.4154	0.715

Our prediction of the branching ratios of the leptonic decays of charmonia and bottomonia based on a size parameter and values from Table 1 and Table 2 is presented in Table 3.

Table 3. Charmonia and bottomonia size parameters and leptonic branching fractions.

Meson	J/ψ $\Lambda_{(J/\psi)}=2.795$ GeV		$\Upsilon(1S)$ $\Lambda_{\Upsilon(1S)}=4.03$ GeV	
	CCQM	Exp.	CCQM	Exp.
$Br(\bar{q}q\to\tau^+\tau^-)$			2.46(7)	2.60(10)
$Br(\bar{q}q\to\mu^+\mu^-)$	5.964(40)	5.961(33)	2.48(7)	2.48(5)
$Br(\bar{q}q\to e^+e^-)$	5.964(40)	5.971(32)	2.48(7)	2.38(11)

Running Constituent Quark Mass for Radial Excitations

To extend the applicability of the Covariant Confined Quark Model (CCQM) to excited quarkonium states, we introduce the concept of a **running constituent quark mass** in the quark loop. The key assumption of this approach is that the *binding energy* of a radially excited quarkonium system $q\bar{q}$ is approximately the same as that of the corresponding ground state V_0.

We define the binding energy of the ground state as

$$E = m_{V0} - 2m_{q0}, \tag{6}$$

where m_{V0} and m_{q0} denote the mass of the ground-state meson and the constituent quark mass, respectively.

For an excited vector meson V', the constituent quark mass $m_{q'}$ is then determined by the condition that the total binding energy remains unchanged:

$$m_{q'} = \frac{m_{V'} - E}{2}. \tag{7}$$

This simple relation ensures a smooth and physically motivated dependence of the constituent quark mass on the quarkonium excitation level, while preserving the internal consistency of the model.

The ground-state quark masses used in this analysis are taken from Table 1. Using these fixed quark masses, the corresponding size parameters Λ_V of the meson vertex functions are determined by fitting the CCQM predictions to the experimental data on leptonic decays $V \to l^+ l^-$.

The obtained values of Λ_V and the predicted leptonic branching fractions for charmonium and bottomonium states are summarized in Tables 4 and Table 5, respectively. The results demonstrate excellent agreement between the CCQM predictions and experimental measurements across all considered decay channels.

Table 4. Charmonia results: size parameters and leptonic branching fractions using a running constituent quark mass.

Meson	*J/ψ* $\Lambda_{(J/\psi)}$=2.795 GeV		*Υ(2S)* $\Lambda_{(J/\psi)}$=0.463 GeV	
	CCQM	Exp.	CCQM	Exp.
$Br(\bar{q}q \to \tau^+ \tau^-)$			0.31(1)	0.31(4)
$Br(\bar{q}q \to \mu^+ \mu^-)$	5.964(40)	5.961(33)	0.81(2)	0.80(6)
$Br(\bar{q}q \to e^+ e^-)$	5.964(40)	5.971(32)	0.81(2)	0.79(2)

Table 5. Bottomonia results: size parameters and leptonic branching fractions using a running constituent quark mass.

Meson	Υ(1S) $\Lambda_{\Upsilon(1S)}$ =4.03 GeV		Υ(2S) $\Lambda_{\Upsilon(2S)}$ =3.77 GeV		Υ(3S) $\Lambda_{\Upsilon(2S)}$ =3.01 GeV	
	CCQM	Exp.	CCQM	Exp.	CCQM	Exp.
$Br(\bar{q}q \to \tau^+ \tau^-)$	2.46(7)	2.60(10)	1.92(0.5)	2.00(21)	2.17(3)	2.29(30)
$Br(\bar{q}q \to \mu^+ \mu^-)$	2.48(7)	2.48(5)	1.93(0.5)	1.93(18)	2.18(3)	2.18(21)
$Br(\bar{q}q \to e^+ e^-)$	2.48(7)	2.38(11)	1.93(0.5)	1.91(16)	2.18(3)	2.18(20)

While the introduction of a running constituent quark mass extends the Covariant Confined Quark Model (CCQM) to describe the mass spectrum and leptonic decays of excited quarkonia, an additional refinement is required to ensure the internal consistency of the model. Specifically, different radial excitations of the same quarkonium family must remain orthogonal, i.e., they cannot transform into one another via a single quark loop. To enforce this physical requirement, we implement an orthogonality condition for the vertex functions of radially excited states.

We represent the vertex function of an n^{th} radial excitation as a polynomial modification of the ground-state vertex function $\Phi_V(-k^2)$:

$$\Phi_n(-k^2) = \left(1 + \sum_{m=1}^{n} c_{m+n-1}\,(s_n k^2)^m\right)\Phi_V(-s_n k^2), \tag{8}$$

where the coefficients c_i are determined by requiring that the corresponding meson amplitudes be mutually orthogonal within the quark loop integral and upper indexes is the degree.

The orthogonality condition is expressed as

$$\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\int \frac{d^4k}{(2\pi)^4 i}\,\Phi_n(-k^2)\,\Phi_m(-k^2)\,\mathrm{tr}\left[\gamma^\mu S\left(k+\frac{1}{2}p\right)\gamma^\nu S\left(k-\frac{1}{2}p\right)\right] = 0, \tag{9}$$

for $n \neq m$, and p is the meson four-momentum. Let us take a precisely look on this relation. There are $\frac{n(n+1)}{2}$ condition for c_{m+n-1} coefficients for $n \geq 0$, where $n=0$ is ground state of quarkonia. Moreover Eq. (9) ensures that the wave functions of distinct radial excitations – such as *J/ψ and Ψ(2S), or ϒ(1S), ϒ(2S), ϒ(3S)* – do not mix under quark-loop transitions. We start by considering a simpler charmonium system. *J/ψ* and *Ψ(2S)* mesons have the following view of vertex functions

$$\begin{aligned} &\Phi_0(-k^2) = \Phi_V(-s_0 k^2) && \text{for } J/\psi \\ &\Phi_1(-k^2) = (1 + c_1\, s_1\, k^2)\Phi_V(-s_1 k^2) && \text{for } \Psi(2S). \end{aligned} \tag{10}$$

Then orthogonality condition Eq. (8) can be rewriting as

$$V_0 V_1 = M_0(s_0, s_1) + c_1 s_1 M_2(s_0, s_1) = 0, \tag{11}$$

here $M_i(s_0, s_1)$ is a matrix element form Eq. (9), where s_0 and s_1 are the parameters of vertex. The low index of M satisfy to power of k^2 before the vertex function.

From this expression, it easily follows that the coefficient $c_1 = -M_0(s_0, s_1)/(s_1 M_2(s_0, s_1))$. Having discussed the procedure of fixing c_1 coefficient for charmonium state, we now turn to a more complex case: the excitations of bottomonium. Here appears 3 complicate structures

$$\begin{aligned} &\Phi_0(-k^2) = \Phi_V(-s_0 k^2) && \text{for } \Upsilon(1S) \\ &\Phi_1(-k^2) = (1 + c_1\, s_1\, k^2)\Phi_V(-s_1 k^2) && \text{for } \Upsilon(2S). \\ &\Phi_2(-k^2) = (1 + c_2\, s_2\, k^2 + c_3\, s_2^2\, k^4)\Phi_V(-s_2 k^2) && \text{for } \Upsilon(3S) \end{aligned} \tag{12}$$

This relations occurs to the following system of equations

$$\begin{aligned} &M_0(s_0,s_1)+\mathrm{c}_1 s_1 M_2(s_0,s_1)=0\\ &M_0(s_0,s_2)+\mathrm{c}_2 s_2 M_2(s_0,s_2)+\mathrm{c}_3 s_2^2 M_4(s_0,s_2)=0\\ &M_0(s_1,s_2)+\mathrm{c}_1 s_1 M_2(s_1,s_2)+\mathrm{c}_2 s_2 M_2(s_1,s_2)+c_1 s_1 \mathrm{c}_2 s_2 M_4(s_1,s_2)\\ &\qquad +\mathrm{c}_3 s_2^2 M_4(s_1,s_2)+c_1 s_1 \mathrm{c}_3 s_2^2 M_6(s_1,s_2)=0. \end{aligned} \tag{13}$$

First equation limits $c_1=-M_0(s_0, s_1)/(s_1 M_2(s_0, s_1))$. Thus, in the system of equation include only two unknown parameters c_2 and c_3. We can obtain them from the following system

$$\begin{aligned} &\mathrm{c}_2 s_2 M_2(s_0,s_2)+\mathrm{c}_3 s_2^2 M_4(s_0,s_2)=-M_0(s_0,s_2)\\ &\mathrm{c}_2\big(s_2 M_2(s_1,s_2)+c_1 s_1 s_2 M_4(s_1,s_2)\big)+\mathrm{c}_3\big(s_2^2 M_4(s_1,s_2)+c_1 s_1 s_2^2 M_6(s_1,s_2)\big)\\ &\qquad =-M_0(s_1,s_2)-\mathrm{c}_1 s_1 M_2(s_1,s_2). \end{aligned} \tag{14}$$

Let us identify the coefficients from these linear equations by using a method so-called Cramer's rule. It is easy to see that we get

$$\begin{aligned} \Delta&=\det\begin{pmatrix} s_2 M_2(s_0,s_2) & s_2^2 M_4(s_0,s_2)\\ s_2 M_2(s_1,s_2)+c_1 s_1 s_2 M_4(s_1,s_2) & s_2^2 M_4(s_1,s_2)+c_1 s_1 s_2^2 M_6(s_1,s_2)\end{pmatrix}\\ \Delta_2&=\det\begin{pmatrix} -M_0(s_0,s_2) & s_2^2 M_4(s_0,s_2)\\ -M_0(s_1,s_2)-c_1 s_1 M_2(s_1,s_2) & s_2^2 M_4(s_1,s_2)+c_1 s_1 s_2^2 M_6(s_1,s_2)\end{pmatrix}\\ \Delta_3&=\det\begin{pmatrix} s_2 M_2(s_0,s_2) & -M_0(s_0,s_2)\\ s_2 M_2(s_1,s_2)+c_1 s_1 s_2 M_4(s_1,s_2) & -M_0(s_1,s_2)-c_1 s_1 M_2(s_1,s_2)\end{pmatrix}. \end{aligned} \tag{15}$$

Finally, after all necessary calculations, all three coefficients can be written in a way that prevents mixing between high- and low-lying excitations

$$c_1=-M_0(s_0,s_1)/(\mathrm{s}_1\, M_2(s_0,s_1)) \quad c_2=\Delta_2/\Delta \quad c_2=\Delta_2/\Delta. \tag{15}$$

Discussion

The calculated branching ratios for charmonia and bottomonia show excellent agreement with experimental data, confirming the model's internal consistency. The implementation of an orthogonality condition for the vertex functions guarantees that high- and low-lying excitations remain independent, avoiding unphysical mixing. Polynomial coefficients are determined via linear systems and Cramer's rule, maintaining predictive power. We reproduce the data for various decays of charmonia and bottomonia by introducing running constituent quark masses. For instance, in case of *Ψ(2S)* meson mass of charm quark is increased to *mc=1.96679 GeV*. Similary, the mass of b-quark rises from m_b*=5.32768 to* m_b*=5.49365* for *Υ(2S)* and *Υ(3S)* mesons, respectively. Numerical value of the coefficients in the orthogonal vertex functions are c_1*=0.802* for *Ψ(2S)* meson, and c_1*=1.979,* c_2*=4.083,* c_3*=2.23*5 for bottomonia.

As one can see the value of parameters and branching ratios of ground of charmnoium and bottomonium were not changed at all. The reason of this if that we used a polynomial modification of the ground-state vertex function in Eq. (8). Therefore, this update does not introduce any changes to the previous results.

Conclusion

We have extended the Covariant Confined Quark Model (CCQM) on the basis of general principles, introducing a running constituent quark mass and orthogonal vertex functions to describe radial excitations of heavy quarkonia. This approach provides a universal mechanism that is not limited to specific states, allowing any quark model built on similar principles to incorporate excited states consistently. By preserving the binding energy of the corresponding ground state and enforcing orthogonality among radial excitations, the model avoids unphysical mixing and maintains internal consistency. These developments significantly enhance the CCQM's applicability to strong and electromagnetic decays of excited hadrons and open the possibility for future studies of hybrid and multiquark systems.

The resulting framework reproduces leptonic decay constants and branching ratios of charmonia and bottomonia in excellent agreement with experimental data using a minimal set of parameters. Beyond quarkonia, this general mechanism can be applied to other hadronic systems, including hybrids and multiquark states, providing a robust and predictive tool for studying excited hadrons from first principles.

Acknowledgement

The research has been funded by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP19678771). The authors gratefully acknowledge Prof. Mikhail A. Ivanov, Prof. Stanislav Dubnička, Dr. Anna Zuzana Dubničková and Dr. Andrej Liptaj for their valuable discussions and insightful suggestions, which significantly contributed to the development of the theoretical framework and the analysis presented in this work.

Authorship contribution statement

Z.Z. Tyulemissov developed the study concept, collected and analyzed data, and drafted the initial manuscript. **A.E. Tyulemissova** interpreted the results, critically revised the text, and approved the final version of the article. Both authors take responsibility for the accuracy of the data and the integrity of all parts of the work.

References

1. G.V.Efimov, M.A.Ivanov and V.E.Lyubovitskij, Strong nucleon and Δ-isobar form factors in quark confinement model, Few Body Syst. 6, no.1, 17–43 (1989), DOI: https://doi.org/10.1007/BF01076281

2. G.Ganbold and M.A.Ivanov, Hidden-charm strong decays of the spin-2 partner of X(3872), Phys. Rev. D 111, no.1, 014007 (2025), DOI: https://doi.org/10.1103/PhysRevD.111.014007, [arXiv:2410.07821 [hep-ph]]

3. M.A.Ivanov, G.Nurbakova and Z.Tyulemissov, Δ Isobar decay in covariant quark model, Phys. Part. Nucl. Lett. 15, no.1, 1–11 (2018), DOI: https://doi.org/10.1134/S1547477118010077

4. T.Gutsche, M.A.Ivanov, J.G.Körner, V.E.Lyubovitskij and K.Xu, Test of the multiquark structure of $a_1(1420)$ in strong two-body decays, Phys. Rev. D 96, no.11, 114004 (2017), DOI: https://doi.org/10.1103/PhysRevD.96.114004, [arXiv:1710.02357 [hep-ph]]

5. T.Gutsche, M.A.Ivanov, J.G.Korner and V.E.Lyubovitskij, Isospin-violating strong decays of scalar single-heavy tetraquarks, Phys. Rev. D 94, no.9, 094012 (2016), DOI: https://doi.org/10.1103/PhysRevD.94.094012, [arXiv:1608.00415 [hep-ph]]

6. T.Gutsche, M.A.Ivanov, J.G.Korner, V.E.Lyubovitskij and P.Santorelli, Light baryons and their electromagnetic interactions in the covariant constituent quark model, Phys. Rev. D 86, 074013 (2012), DOI: https://doi.org/10.1103/PhysRevD.86.074013, [arXiv:1207.7052 [hep-ph]]

7. C.T.Tran, M.A.Ivanov, N.D.Nguyen and Q.C.Vo, Polarization observables in semileptonic V → P decays of quarkonia, J. Phys. Conf. Ser. 3040, no.1, 012003 (2025), DOI: https://doi.org/10.1088/1742-6596/3040/1/012003, [arXiv:2506.21902 [hep-ph]]

8. C.T.Tran, M.A.Ivanov, P.Santorelli and H.C.Tran, Study of the semileptonic decays Υ(1S) → B_(c) ℓ ν̄_ℓ, Chin. Phys. C 49, no.1, 013111 (2025), DOI: https://doi.org/10.1088/1674-1137/ad83ab, [arXiv:2408.13776 [hep-ph]]

10. S.Groote, M.A.Ivanov, J.G.Körner, V.E.Lyubovitskij, P.Santorelli and C.T.Tran, Form-factor-independent test of lepton universality in semileptonic heavy meson and baryon decays, Phys. Rev. D 103, no.9, 093001 (2021), DOI: https://doi.org/10.1103/PhysRevD.103.093001, [arXiv:2102.12818 [hep-ph]]

11. T.Gutsche, M.A.Ivanov, J.G.Körner, V.E.Lyubovitskij and Z.Tyulemissov, Analysis of the semileptonic and nonleptonic two-body decays of the double heavy charm baryon states Ξ_cc^++, Ξ_cc^+ and Ω_cc^+, Phys. Rev. D 100, no.11, 114037 (2019), DOI: https://doi.org/10.1103/PhysRevD.100.114037, [arXiv:1911.10785 [hep-ph]]

12. T.Gutsche, M.A.Ivanov, J.G.Körner, V.E.Lyubovitskij and Z.Tyulemissov, Ab initio three-loop calculation of the W-exchange contribution to nonleptonic decays of double charm baryons, Phys. Rev. D 99, no.5, 056013 (2019), DOI: https://doi.org/10.1103/PhysRevD.99.056013, [arXiv:1812.09212 [hep-ph]]

13. T.Gutsche, M.A.Ivanov, J.G.Körner and V.E.Lyubovitskij, Nonleptonic two-body decays of single heavy baryons Λ_Q, Ξ_Q and Ω_Q (Q=b,c) induced by W emission in the covariant confined quark model, Phys. Rev. D 98, no.7, 074011 (2018), DOI: https://doi.org/10.1103/PhysRevD.98.074011, [arXiv:1806.11549 [hep-ph]]

14. K.Raya, M.A.Bedolla, J.J.Cobos-Martínez and A.Bashir, Heavy quarkonia in a contact interaction and an algebraic model: mass spectrum, decay constants, charge radii and elastic and transition form factors, Few Body Syst. 59, no.6, 133 (2018), DOI: https://doi.org/10.1007/s00601-018-1455-y, [arXiv:1711.00383 [nucl-th]]

15. N.R.Soni, B.R.Joshi, R.P.Shah, H.R.Chauhan and J.N.Pandya, Q\bar{Q} (Q∈{b,c}) spectroscopy using the Cornell potential, Eur. Phys. J. C 78, no.7, 592 (2018), DOI: https://doi.org/10.1140/epjc/s10052-018-6068-6, [arXiv:1707.07144 [hep-ph]]

16. Y.Li, P.Maris and J.P.Vary, Quarkonium as a relativistic bound state on the light front, Phys. Rev. D 96, 016022 (2017), DOI: https://doi.org/10.1103/PhysRevD.96.016022, [arXiv:1704.06968 [hep-ph]]

17. J.Adam and al. [ALICE], Inclusive quarkonium production at forward rapidity in pp collisions at √s=8 TeV, Eur. Phys. J. C 76, no.4, 184 (2016), DOI: https://doi.org/10.1140/epjc/s10052-016-3987-y, [arXiv:1509.08258 [hep-ex]]

Ж.Ж.Тюлемисов*, А.Е.Тюлемисова
Біріккен ядролық зерттеулер институты, Дубна, Ресей
*(E-mail: *zhomart.tyulemissov@gmail.com, akmaral.tyulemissova@gmail.com)*

Ауыр кварконийлардың радиалды қозған күйлерін сипаттау үшін Ковариантты Шектелген Кварк Моделін кеңейту

Аңдатпа. Біз ауыр кварконийлардың радиалды қозған күйлерін үйлесімді сипаттауға мүмкіндік беретін Ковариантты Шектелген Кварк Моделінің (КШКМ) кеңейтілген нұсқасын ұсынамыз. Тұрақты құрамдас кварк массаларына негізделген дәстүрлі КШКМ формуласы құрамдас массалардың квадраттарының қосындысынан артық квадраттық массаға ие күйлерді сипаттауда шектеулерге тап болады. Бұл мәселені шешу үшін біз кварк ілмегінде жүрмелі құрамдас кварк массасын енгіздік, бұл қозған адрондық ыдырауларды дәл сипаттауға мүмкіндік береді. Қозған күйдің байланыс энергиясы сәйкес негізгі күйдің байланыс энергиясына тең деп қабылданады, бұл кварконий мен олардың құрамдастары арасындағы қарапайым массалық қатынасты тудырады. Модель параметрлері чармония мен боттомонияның лептондық ыдыраулары бойынша эксперименттік деректер негізінде анықталып, бақылаулармен тамаша сәйкестік көрсетеді. Сонымен қатар, радиалды қозған күйлердің ортогоналдық шарты модификацияланған вершиналық функциялар арқылы жүзеге асырылады, бұл әртүрлі қозған күйлердің араласпауын қамтамасыз етеді. Бұл жетілдіру КШКМ моделінің ішкі үйлесімділігін және қозған адрондық жүйелерді сипаттаудағы болжамдық қабілетін арттырады.

Түйін сөздер: ковариантты шектелген кварк моделі, радиалды қозған күйлер, жүрмелі құрамдас кварк массасы, кварконий, чармоний, боттомоний, лептондық ыдырау.

Ж.Ж. Тюлемисов*, А.Е. Тюлемисова
Объединенный институт ядерных исследований, Дубна, Российская Федерация
*(E-mail: *zhomart.tyulemissov@gmail.com, akmaral.tyulemissova@gmail.com)*

Расширение ковариантной модели кварков для описания радиальных возбуждений тяжёлых кварткониев

Аннотация. Мы представляем расширение Ковариантной Модели Кварков (КМК), которое обеспечивает согласованное описание радиальных возбуждённых состояний тяжёлых кварткониев. Традиционная формулировка КМК, основанная на статических массах составных кварков, сталкивается с ограничениями при описании состояний, квадрат массы которых превышает сумму квадратов составных масс. Чтобы преодолеть это, мы вводим бегущую массу составного кварка в кварковой петле, что позволяет точно описывать распады возбуждённых адронов. Энергия связи возбуждённого состояния предполагается равной энергии связи соответствующего основного состояния, что приводит к простой зависимости масс между кварконием и его составными частицами. Параметры модели определяются из экспериментальных данных лептонных распадов чармония и боттомония, демонстрируя отличное согласие с наблюдаемыми.

Кроме того, было использовано условие ортогональности для модифицированных вершинных функций радиальных возбуждений, что обеспечивает отсутствие смешивания различных возбуждённых состояний. Это усовершенствование повышает внутреннюю согласованность и предсказательную способность КМК при описании возбуждённых адронных систем.

Ключевые слова: ковариантная модель кварков, радиальные возбуждения, бегущая масса составного кварка, кварконий, чармоний, боттомоний, лептонный распад.

References

1. G.V.Efimov, M.A.Ivanov and V.E.Lyubovitskij, Strong nucleon and Δ-isobar form factors in quark confinement model, Few Body Syst. 6, no.1, 17–43 (1989), DOI: https://doi.org/10.1007/BF01076281

2. G.Ganbold and M.A.Ivanov, Hidden-charm strong decays of the spin-2 partner of X(3872), Phys. Rev. D 111, no.1, 014007 (2025), DOI: https://doi.org/10.1103/PhysRevD.111.014007, [arXiv:2410.07821 [hep-ph]]

3. M.A.Ivanov, G.Nurbakova and Z.Tyulemissov, Δ Isobar decay in covariant quark model, Phys. Part. Nucl. Lett. 15, no.1, 1–11 (2018), DOI: https://doi.org/10.1134/S1547477118010077

4. T.Gutsche, M.A.Ivanov, J.G.Körner, V.E.Lyubovitskij and K.Xu, Test of the multiquark structure of a_1(1420) in strong two-body decays, Phys. Rev. D 96, no.11, 114004 (2017), DOI: https://doi.org/10.1103/PhysRevD.96.114004, [arXiv:1710.02357 [hep-ph]]

5. T.Gutsche, M.A.Ivanov, J.G.Korner and V.E.Lyubovitskij, Isospin-violating strong decays of scalar single-heavy tetraquarks, Phys. Rev. D 94, no.9, 094012 (2016), DOI: https://doi.org/10.1103/PhysRevD.94.094012, [arXiv:1608.00415 [hep-ph]]

6. T.Gutsche, M.A.Ivanov, J.G.Korner, V.E.Lyubovitskij and P.Santorelli, Light baryons and their electromagnetic interactions in the covariant constituent quark model, Phys. Rev. D 86, 074013 (2012), DOI: https://doi.org/10.1103/PhysRevD.86.074013, [arXiv:1207.7052 [hep-ph]]

7. C.T.Tran, M.A.Ivanov, N.D.Nguyen and Q.C.Vo, Polarization observables in semileptonic V → P decays of quarkonia, J. Phys. Conf. Ser. 3040, no.1, 012003 (2025), DOI: https://doi.org/10.1088/1742-6596/3040/1/012003, [arXiv:2506.21902 [hep-ph]]

8. C.T.Tran, M.A.Ivanov, P.Santorelli and H.C.Tran, Study of the semileptonic decays ϒ(1S) → B_(c) ℓ ν̄_ℓ, Chin. Phys. C 49, no.1, 013111 (2025), DOI: https://doi.org/10.1088/1674-1137/ad83ab, [arXiv:2408.13776 [hep-ph]]

9. S.Groote, M.A.Ivanov, J.G.Körner, V.E.Lyubovitskij, P.Santorelli and C.T.Tran, Form-factor-independent test of lepton universality in semileptonic heavy meson and baryon decays, Phys. Rev. D 103, no.9, 093001 (2021), DOI: https://doi.org/10.1103/PhysRevD.103.093001, [arXiv:2102.12818 [hep-ph]]

10. T.Gutsche, M.A.Ivanov, J.G.Körner, V.E.Lyubovitskij and Z.Tyulemissov, Analysis of the semileptonic and nonleptonic two-body decays of the double heavy charm baryon states Ξ_cc^++, Ξ_cc^+ and Ω_cc^+, Phys. Rev. D 100, no.11, 114037 (2019), DOI: https://doi.org/10.1103/PhysRevD.100.114037, [arXiv:1911.10785 [hep-ph]]

11. T.Gutsche, M.A.Ivanov, J.G.Körner, V.E.Lyubovitskij and Z.Tyulemissov, Ab initio three-loop calculation of the W-exchange contribution to nonleptonic decays of double charm baryons, Phys. Rev. D 99, no.5, 056013 (2019), DOI: https://doi.org/10.1103/PhysRevD.99.056013, [arXiv:1812.09212 [hep-ph]]

12. T.Gutsche, M.A.Ivanov, J.G.Körner and V.E.Lyubovitskij, Nonleptonic two-body decays of single heavy baryons Λ_Q, Ξ_Q and Ω_Q (Q=b,c) induced by W emission in the covariant confined quark model, Phys. Rev. D 98, no.7, 074011 (2018), DOI: https://doi.org/10.1103/PhysRevD.98.074011, [arXiv:1806.11549 [hep-ph]]

13. K.Raya, M.A.Bedolla, J.J.Cobos-Martínez and A.Bashir, Heavy quarkonia in a contact interaction and an algebraic model: mass spectrum, decay constants, charge radii and elastic and transition form factors, Few Body Syst. 59, no.6, 133 (2018), DOI: https://doi.org/10.1007/s00601-018-1455-y, [arXiv:1711.00383 [nucl-th]]

14. N.R.Soni, B.R.Joshi, R.P.Shah, H.R.Chauhan and J.N.Pandya, Q\bar{Q} (Q∈{b,c}) spectroscopy using the Cornell potential, Eur. Phys. J. C 78, no.7, 592 (2018), DOI: https://doi.org/10.1140/epjc/s10052-018-6068-6, [arXiv:1707.07144 [hep-ph]]

15. Y.Li, P.Maris and J.P.Vary, Quarkonium as a relativistic bound state on the light front, Phys. Rev. D 96, 016022 (2017), DOI: https://doi.org/10.1103/PhysRevD.96.016022, [arXiv:1704.06968 [hep-ph]]

16. J.Adam and al. [ALICE], Inclusive quarkonium production at forward rapidity in pp collisions at $\sqrt{s}=8$ TeV, Eur. Phys. J. C 76, no.4, 184 (2016), DOI: https://doi.org/10.1140/epjc/s10052-016-3987-y, [arXiv:1509.08258 [hep-ex]]

Information about the authors:

***Tyulemissov Z.Z.* –** corresponding author, PhD in Physics and Mathematics, Researcher of The Joint Institute for Nuclear Research, 6 Joliot Curie str., 141980, Dubna, Russia

***Tyulemissova A.E.* –** master in physics, Junior Researcher of The Joint Institute for Nuclear Research, 6 Joliot Curie str., 141980, Dubna, Russia

***Тюлемисов Ж.Ж.* –** автор для корреспонденции, кандидат физико-математических наук, научный сотрудник Объединенного института ядерных исследований, ул. Жолио Кюри, 6, 141980, Дубна, Россия

***Тюлемисова А.Е.* –** магистр, младший научный сотрудник Объединенного института ядерных исследований, ул. Жолио Кюри, 6, 141980, Дубна, Россия

Тюлемисов Ж.Ж. – хат-хабар авторы, физика-математика ғылымдарының кандидаты, Біріккен ядролық зерттеулер институтының ғылыми қызметкері, Жолио-Кюри көшесі, 6, 141980, Дубна, Ресей

Тюлемисов А.Е. – магистр, Біріккен ядролық зерттеулер институтының кіші ғылыми қызметкері, Жолио-Кюри көшесі, 6, 141980, Дубна, Ресей