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Constraining power-law cosmology with observational Hubble data

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Abstract. In this work, we analyze observational constraints on power-law cosmology, a simple yet insightful model of the universe's expansion. This framework is primarily characterized by two parameters: the Hubble constant (H_0), which sets the present expansion rate, and the deceleration parameter (q), which describes the nature of cosmic acceleration or deceleration. By employing the latest compilation of 31 observational Hubble data points, we place bounds on these parameters with improved accuracy. Our analysis shows that power-law cosmology is capable of providing a remarkably good fit to the data, with the obtained 1σ estimates remaining consistent with recent results in the literature. This agreement highlights the relevance of the model as a phenomenological description of cosmic expansion. However, despite its success in reproducing large-scale trends, the model has limitations, particularly in addressing the finer dynamical features, such as the transition from deceleration to acceleration in cosmic history. Overall, it is shown that although power-law cosmology provides an elegant and phenomenologically viable description of many aspects of cosmic evolution, it remains insufficient to solve all observational problems, in particular the dynamical transition of cosmic acceleration.

Keywords: Cosmic acceleration, data analysis, Hubble data, redshift, power-law cosmology

Introduction

The Standard Cosmological Model (SM) of the Universe, namely the ΛCDM framework supplemented with an inflationary phase, has been remarkably successful in explaining a wide range of observations. Nevertheless, the cosmological constant problem continues to be one of the major unresolved issues of modern cosmology [1], motivating the exploration of alternative models. Among these, power-law cosmology offers an interesting framework to address several classical problems of the standard model, including the age, flatness, and

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horizon problems. In this approach, the cosmic expansion is described by the scale factor , with as a positive constant. Scenarios with $\beta \approx 1$ have been studied extensively in different contexts [2-9], while phantom power-law cosmology has been explored in Ref. [10]. Notably, power-law cosmology naturally avoids the horizon and flatness problems, accommodates high-redshift objects (mitigating the age problem), and offers dynamical approaches to the fine-tuning issue associated with the cosmological constant [11-15]. A power-law expansion with provides an excellent fit to a variety of cosmological observations. Models supporting such a coasting evolution are falsifiable through classical cosmological tests, as they exhibit clear and verifiable predictions. Tests such as galaxy number counts versus redshift and angular diameter distance measurements [16] favor this type of evolution, though their reliability is limited by evolutionary effects (e.g., mergers). By contrast, Type Ia supernovae, being robust standard candles, and $H(z)$ measurements have emerged as far more reliable probes.

Cosmological parameters form the backbone of any model, as their precise determination is essential for describing the present cosmic expansion. In particular, the Hubble constant (H_0), which measures the current expansion rate, and the deceleration parameter (q), which characterizes its acceleration or deceleration, are of central importance. In recent years, numerous efforts have been devoted to constraining H_0. For instance, Freedman et al. [17] obtained $H_0 = 72 \pm 8$ km/s/Mpc, while the most recent Planck results give $H_0 = 67.4 \pm 0.5$ km/s/Mpc [18]. In addition, several works have constrained H_0, q, and β in open, closed, and flat power-law cosmologies. Numerical results for the flat case are summarized in Table 2. Recently, Rani et. al. [19] investigated observational constraints on power-law cosmology using $H(z)$ and SN Ia data, providing a detailed discussion of its viability. In the present work, we perform a similar analysis for the flat case, employing the most recent datasets of $H(z)$ measurements [20]. The structure of this paper is as follows: Section 2 outlines the mathematical framework of the power-law cosmology ansatz. Section 3 discusses the methodology used to constrain the model's power index with the $H(z)$ data set. Finally, Section 4 presents a summary of the main results.

Mathematical Framework of Power-Law Cosmology

The observed large-scale homogeneity and isotropy of the universe indicate that its geometry can be described by the Friedmann–Lemaître–Robertson–Walker (FLRW) metric. For a spatially flat FLRW metric, the line element is given by

$$ds^2 = c^2 dt^2 - a^2(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right], \qquad (2.1)$$

Here, c denotes the speed of light, t represents the proper time, and (r, θ, ϕ) are the spherical polar co-moving coordinates. The function $a(t)$ is known as the cosmic scale factor or expansion parameter that describes the time evolution of the universe. In this work, we consider the general framework of power-law cosmology, where the scale factor evolves as

$$a(t)=a_0\left(\frac{t}{t_0}\right)^{\beta}, \tag{2.2}$$

with t_0 denoting the present age of the Universe and β a dimensionless positive constant. Here and throughout, the subscript 0 refers to the present-day values of cosmological parameters. The Hubble parameter in this model is

$$H=\frac{\dot{a}}{a}=\frac{\beta}{t}, H_0=\frac{\beta}{t_0}. \tag{2.3}$$

The relation between the scale factor and the redshift is

$$\frac{a(t)}{a_0}=\frac{1}{1+z}. \tag{2.4}$$

Accordingly, the age of the Universe at redshift z can be written as

$$t(z)=\frac{\beta}{H(z)}, \tag{2.5}$$

where

$$H(z)=H_0(1+z)^{1/\beta}. \tag{2.6}$$

The acceleration of the Universe is quantified by the deceleration parameter q, defined as

$$q=-\frac{\ddot{a}}{aH^2}=\frac{1}{\beta}-1, \tag{2.7}$$

where $q<0$ indicates an accelerating universe, while $q\geq 0$ corresponds to a decelerating or coasting expansion. In terms of q, Eq. (2.6) can be recast as

$$H(z)=H_0(1+z)^{(1+q)}. \tag{2.8}$$

Equation (2.8) highlights that the parameters H_0 and q govern the cosmic history in power-law cosmology. In this paper, we focus on constraining these two parameters using the latest 31 data points of $H(z)$ measurements .

Table 1: Hubble parameter measurements at different redshifts [20]

0.0900	69.0	12.0
0.1700	83.0	8.0
0.2700	77.0	14.0
0.4000	95.0	17.0
0.9000	117.0	23.0
1.3000	168.0	17.0

1.4300	177.0	18.0
1.5300	140.0	14.0
1.7500	202.0	40.0
0.4800	97.0	62.0
0.8800	90.0	40.0
0.1791	75.0	4.0
0.1993	75.0	5.0
0.3519	83.0	14.0
0.5929	104.0	13.0
0.6797	92.0	8.0
0.7812	105.0	12.0
0.8754	125.0	17.0
1.0370	154.0	20.0
0.0700	69.0	19.6
0.1200	68.6	26.2
0.2000	72.9	29.6
0.2800	88.8	36.6
1.3630	160.0	33.6
1.9650	186.5	50.4
0.3802	83.0	13.5
0.4004	77.0	10.2
0.4247	87.1	11.2
0.4497	92.8	12.9
0.4783	80.9	9.0
0.4700	89.0	23.0

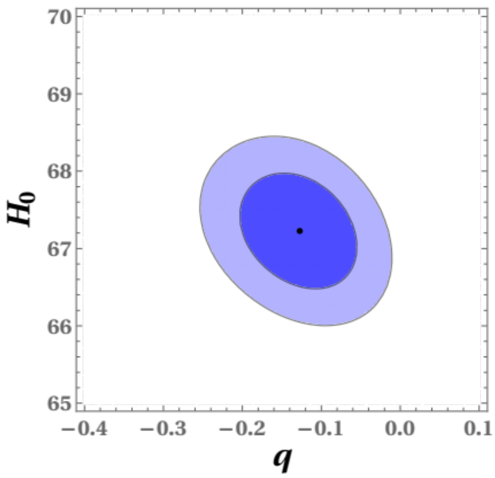

Figure 1. Likelihood contours in the q–H_0 plane obtained from H(z) data. The 1σ (dark shaded) and 2σ (light shaded) confidence regions are shown, with H_0 in units of km/s/Mpc. The black dot marks the best-fit values of q and H_0.

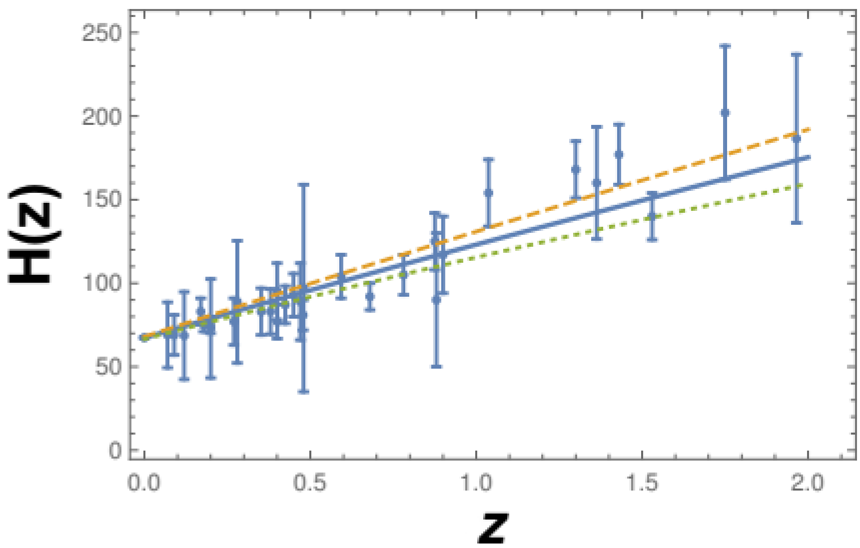

Figure 2. Observational H(z) data points with error bars. The solid curve represents the best-fit model, while the dashed and dotted curves correspond to the maximum and minimum values within the 1σ region. The fit shows excellent agreement with the observational data, particularly at redshifts z < 1. The H(z) is expressed in units of km/s/Mpc.

Data Analysis: Observational Hubble data

We constrain the parameters H_0 and q using the latest 31 data points of $H(z)$ measurements in the redshift range $0.07 \le z \le 1.75$, see Table 1. To complete the dataset, we additionally include the most precise determination of the Hubble constant from the *Planck* results. The results are summarized in Table 2. The X^2 function is defined as

$$\chi_H^2(q, H_0) = \sum_{j=1}^{31} \frac{(H_{exp}(z_j; q, H_0) - H_{obs}(z_j))^2}{\sigma_j^2} \tag{3.1}$$

where H_{exp} is the theoretical prediction from power-law cosmology, H_{obs} the observed value, and σ_j the corresponding 1σ error. The model involves two free parameters, q and H_0. Since power-law cosmology requires $\beta > 0$, the parameter space is restricted to $q > -1$ and $H_0 \ge 0$. Minimizing the X^2 function yields the best-fit values

$$q = -0.1273, H_0 = 67.2264 \text{ km/s/Mpc}, \chi_\delta^2 = 0.6404,$$

where $\chi_\delta^2 = \chi_{min}^2/\text{d.o.f}$. The 1σ confidence intervals are found to be

$$q = -0.1273^{+0.0724}_{-0.0760}, H_0 = 67.2264^{+0.7471}_{-0.7469} \text{km/s/Mpc}.$$

These results demonstrate that the power-law cosmological model is consistent with the most recent $H(z)$ measurements and supports an accelerating universe, in agreement with other independent observations. The 1σ (dark shaded) and 2σ (light shaded) likelihood contours in the q_H_0 plane are shown in Fig. 1, with the black dot marking the best-fit

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point. Fig. 2 exhibits the observational $H(z)$ data points with error bars. The solid curve represents the best-fit model, while the dashed and dotted curves correspond to the maximum and minimum values within the 1σ region. The fit shows excellent agreement with the observational data, particularly at redshifts $z<1$.

Table 2: Summary of the numerical results for a flat universe

Data	q	H_0 (km|s|Mpc)	Reference
$H(z)$ (31 points)	$-0.1273^{+0.0.0724}_{-0.0760}$	$67.2264^{+0.7471}_{-0.7469}$	This work
$H(z)$ (14 points)	$-0.18^{+0.12}_{-0.12}$	$67.2264^{+0.7471}_{-0.7469}$	[21]
$H(z)$ (29 points)	$-0.0440^{+0.0496}_{-0.0508}$	$65.1738^{+1.6035}_{-1.5990}$	[19]

Conclusion

Precision cosmological observations provide a powerful tool for probing the fundamental properties of the Universe. In this work, we have examined power-law cosmology, characterized by $a(t)\propto t^{\beta}$, which exhibits several distinctive features. For $\beta\geq 1$, this model naturally addresses the horizon, flatness, and age problems , and offers a framework to dynamically alleviate the cosmological constant problem. We have constrained the key cosmological parameters, the Hubble constant H_0 and the deceleration parameter q, using the latest $H(z)$ observations. A notable advantage of this model is its simplicity, requiring only two free parameters to fit the data. The resulting numerical constraints are summarized in Tables 2. Our analysis indicates a negative value of the deceleration parameter, consistent with an accelerating Universe, confirming that $H(z)$ is well described by the power-law model. The estimated values of H_0 agree closely with independent determinations reported in the literature, as discussed in Section 1. Contour and error bar plots (Figs. 1 and 2) further demonstrate the consistency of the best-fit model with observational data. Despite its several appealing features, power-law cosmology is limited in capturing the redshift-dependent transition from deceleration to acceleration, as the deceleration parameter q is constant in this model. In summary, while power-law cosmology provides an elegant and phenomenologically viable description of many aspects of cosmic evolution, it remains insufficient to address all observational challenges, particularly the dynamical transition of cosmic acceleration.

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The contribution of the authors

Rida Fatima – Performed the theoretical part of the manuscript.

Mohd Shahalam – Developed the research idea and framework. Performed the data analysis part, wrote the manuscript, and supervised the project.

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Хаббл телескопының бақылау деректерімен дәрежелік космологиялық модельді шектеу

Аңдатпа. Бұл жұмыста біз космологияның бақылаушы шектеулерін талдаймыз-ғаламның кеңеюінің қарапайым, бірақ мағыналы моделі. Бұл модель екі параметрмен сипатталады: Қазіргі кеңею жылдамдығын белгілейтін Хаббл константасы (H_0) және ғарыштық үдеудің немесе баяулаудың табиғатын сипаттайтын баяулау параметрі (q). Хаббլдың 31 бақылау нүктесінің соңғы жинағын қолдана отырып, біз осы параметрлерге жоғары дәлдікпен шектеулер қоямыз. Біздің талдауымыз көрсеткендей, космология деректердің таңқаларлықтай жақсы сәйкестігін қамтамасыз ете алады, алынған 1σ бағалары әдебиетте жарияланған соңғы нәтижелерге сәйкес келеді. Бұл сәйкестік ғарыштық кеңеюдің феноменологиялық сипаттамасы ретінде модельдің өзектілігін көрсетеді. Дегенмен, оның ауқымды трендтерді сәтті қайталауына қарамастан, модельде шектеулер бар, әсіресе тарихтағы баяулаудан үдеуге көшу сияқты нәзік динамикалық сипаттамаларды қарастырған кезде. Жалпы, көрсетілген космология ғарыштық эволюцияның көптеген аспектілерін талғампаз және феноменологиялық тұрғыдан тиімді сипаттауды қамтамасыз еткенімен, ол барлық бақылау мәселелерін, атап айтқанда ғарыштық үдеудің динамикалық ауысуын шешу үшін әлі де жеткіліксіз.

Түйін сөздер: ғарыштық жеделдету, деректерді талдау, Хаббл деректері, қызылға жылжу, қуат заңы космологиясы

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Ограничение степенной космологической модели данными наблюдений телескопа Хаббла

Абстракт. В данной работе мы анализируем наблюдательные ограничения степенной космологии – простой, но содержательной модели расширения Вселенной. Эта модель характеризуется двумя параметрами: постоянной Хаббла (H_0), которая задаёт современную скорость расширения, и параметром замедления (q), который описывает природу космического ускорения или замедления. Используя последнюю компиляцию из 31 точки наблюдений Хаббла, мы устанавливаем ограничения на эти параметры с повышенной точностью. Наш анализ показывает, что степенная космология способна обеспечить удивительно хорошее соответствие данным, при этом полученные оценки 1σ остаются согласованными с недавними результатами, опубликованными в литературе. Это соответствие подчёркивает актуальность модели как феноменологического описания космического расширения. Однако, несмотря на её успешное воспроизведение крупномасштабных трендов, модель имеет ограничения, особенно при рассмотрении более тонких динамических характеристик, таких как переход от замедления к ускорению в истории. В целом показано что, хотя степенная космология обеспечивает элегантное и феноменологически жизнеспособное описание многих аспектов космической эволюции, ее по-прежнему недостаточно для решения всех наблюдательных задач, в частности динамического перехода космического ускорения.

Ключевые слова: космическое ускорение, анализ данных, данные Хаббла, красное смещение, степенная космология

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