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Gravitational Redshift in Generalized Gravity Models

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Abstract. This paper explores the phenomenon of gravitational redshift within the framework of extended gravity models, which go beyond the conventional Einsteinian General Relativity. The gravitational redshift is a key observable manifestation of spacetime curvature and serves as an important tool for testing theories of gravity. In the framework of general relativity (GR), it is explained as a consequence of time dilation in a gravitational field. This paper examines the features of the manifestation of gravitational redshift in various generalized gravitational theories and conducts a comparative analysis of the time predictions of these models with the results of observations, in particular, near compact objects and on cosmological scales. We establish a relationship between the redshift Z and the normalized scale factor $\alpha(\iota)$ of the universe, linking it through the Hubble parameter $\mathrm{H}(\mathrm{T})$. We develop both analytical and numerical approaches to examine the time evolution of redshift. By solving the governing equations, we derive expressions for $Z, \alpha(\mathrm{T})$, and their derivatives concerning time. The results align with current cosmological observations and provide new insights into the dynamics of the universe's expansion. Additionally, visualizations of the evolution of the scale factor, its first and second derivatives over time, are presented. These findings contribute to a better understanding of the complex interaction between gravitational forces and cosmic expansion.
Keywords: Gravitational redshift; scale factor; Hubble parameter; cosmological models; generalized gravity.

Introduction

Perhaps one of the most important observable parameters of all cosmological objects is the so-called redshift, based on which a conclusion is drawn about the expansion of the universe. The essence of this phenomenon in cosmology is the shift of the emission spectrum lines of

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luminous objects towards longer wavelengths. It is known that excited atoms of rarefied gases or vapors, which can occur when any chemical element is heated, emit light, the decomposition of which on a prism forms a linear spectrum consisting of separate colored lines. At the same time, each chemical element has a linear spectrum characteristic of it. This is due to the fact that the atoms of such elements, isolated from each other, emit light only of certain wavelengths. These waves have strictly defined resonant frequencies, which in special instruments – spectroscopes – are visible as dark or light lines in certain parts of the spectrum characteristic of this substance. The shift of these initial spectral lines of chemical elements towards longer wavelengths, towards the "red" side, is called redshift. In cosmology, redshifts are denoted by z and defined as a relative increase in wavelength $z = \frac{\lambda_0 - \lambda}{\lambda}$.More generally, this equation is written as follows $1 + z = \frac{a(t_0)}{a(t)} = \frac{\nu}{\nu_0} = \frac{\lambda}{\lambda_0}$. All values marked with the index 0 refer to the moment of wave reception . Since in the expanding universe, the wavelength of the received signal is longer than the emitted one. The value called the redshift parameter is equal to the relative increase in the wavelength of the received electromagnetic signal. The magnitude of the redshift depends on the relative speed of the objects – the transmitter, the wave generator, and the receiver, so the redshift allows you to determine this relative speed [5-6]. Gravitational redshift is a critical observational effect in cosmology, indicating the interaction between light and the gravitational field of an expanding universe [1]. In standard cosmological models, this phenomenon is typically described by the relation between redshift z, scale factor $\alpha(t)$, and the Hubble parameter. However, more general gravity theories, which extend the framework of General Relativity, present new opportunities for exploring redshift in a broader context.

This study aims to derive both analytical and numerical solutions for the evolution of redshift within generalized gravity models. By considering a normalized scale factor, we analyze how redshift depends on time and the Hubble parameter, providing a detailed examination of the universe's expansion [2].

Theoretical Framework

The FLRW metric:

$$ds^2 = -dt^2 + a(t)^2[dx^2 + dy^2 + dz^2] \tag{1}$$

$$X = -\frac{1}{2}\dot{\phi}^2 \tag{2}$$

$$R = 6\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right) \tag{3}$$

$F(R, X) = R + \alpha X^2 + \beta R^2$ – includes a quadratic curvature (as in the old inflationary models of the Starobinsky type) and a nonlinear kinetic term characterized by a nonlinear k – *essence.*

Connection with redshift:

$$1 + z = \frac{a_0}{a(t)} \tag{4}$$

It is possible to express z(t) and study its evolution depending on the solutions of the equations flowing into the model f (R, X).

The normalized scale factor $\alpha(t)$ is connected to the redshift z through the following equation:

$$1 + z = \frac{a_0}{a}, \tag{5}$$

From the definition of the Hubble parameter:

$$H = \frac{1}{H_0(t - t_0)} \tag{6}$$

This simplifies to:

$$dt = -\frac{dz}{(1+z)H(t)} \tag{7}$$

For simplicity, we assume a time-dependent Hubble parameter, which allows the equation to be solved numerically. Upon integrating, we obtain expressions for and, providing a time-evolving model for the redshift [3].

$$H = \frac{1}{H_0(t - t_0)}, \tag{8}$$

$$dt = -\frac{dz}{(1+z)H(t)} = -\frac{dz}{(1+z)\times\frac{1}{H_0(t-t_0)}} \tag{9}$$

Integrating Time and Redshift. By integrating both sides, we derive the explicit relationship between time and redshift:

$$\int \frac{dt}{(t-t_0)} = -H_0 \int \frac{dz}{1+z} \tag{10}$$

$$\int \frac{dt}{(t-t_0)} = ln|t - t_0| + C_1 \tag{11}$$

$$\int \frac{dz}{1+z} = ln|1 + z| + C_2 \tag{12}$$

The integrations yield:

$$\ln|t\text{-}t_0| = -H_0 \ln|1+z| + C \tag{13}$$

$$ln|1 + z| = -\frac{ln|t-t_0| - C}{H_0} \tag{14}$$

where C is the constant of integration. Rearranging this, we have:

$$|1 + z| = (t - t)^{-\frac{1}{H_0}} \times e^{-\frac{C}{H_0}} \tag{15}$$

The Final Formula for Redshift. The redshift z can now be expressed as:

$$z = \left((t - t_0)^{H_0} \times e^{-\frac{C}{H_0}}\right) - 1 \tag{16}$$

Results and discussion

This equation directly relates redshift z to the time t, considering the universe's expansion. We can derive the evolution equation for the redshift:

Physical Interpretation. This formula explains how the wavelength of light changes as the universe expands. In generalized gravity models such as $f(R, X)$, it plays a crucial role in describing the geometry and evolution of the universe.

The parameter H_0 defines the rate of expansion, while C corresponds to the initial conditions. The redshift z encodes the observational evidence of this expansion through light emitted by distant objects [4-8].

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Numerical Results and Graphical Representations

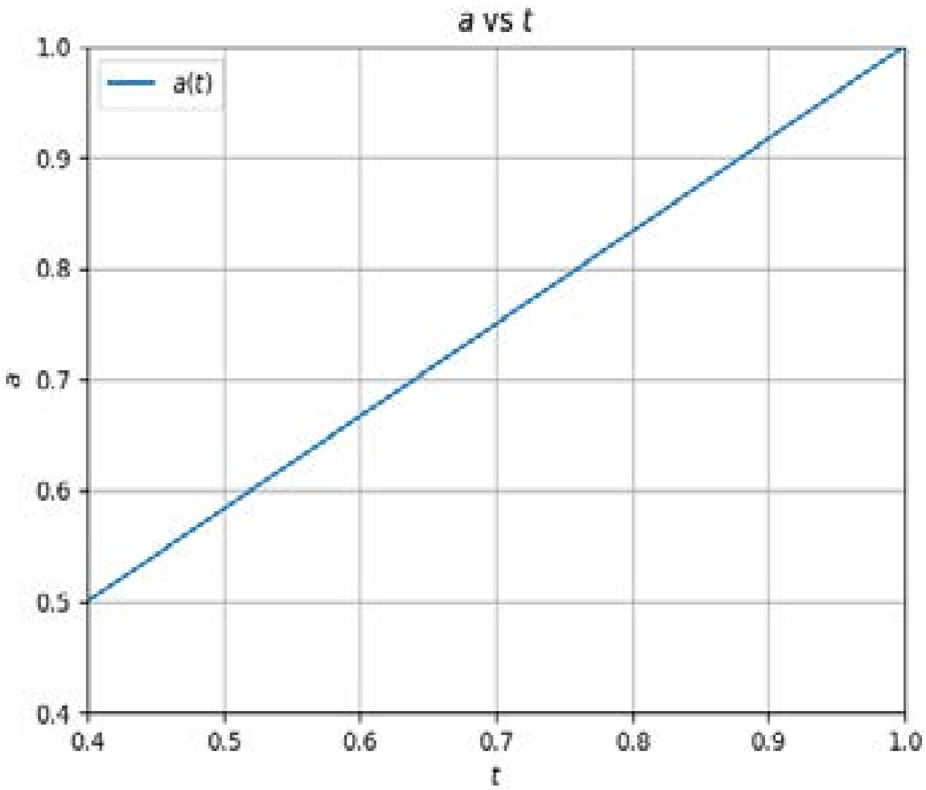

Figure 1. illustrates the linear increase of $a(t)$, consistent with an accelerating universe

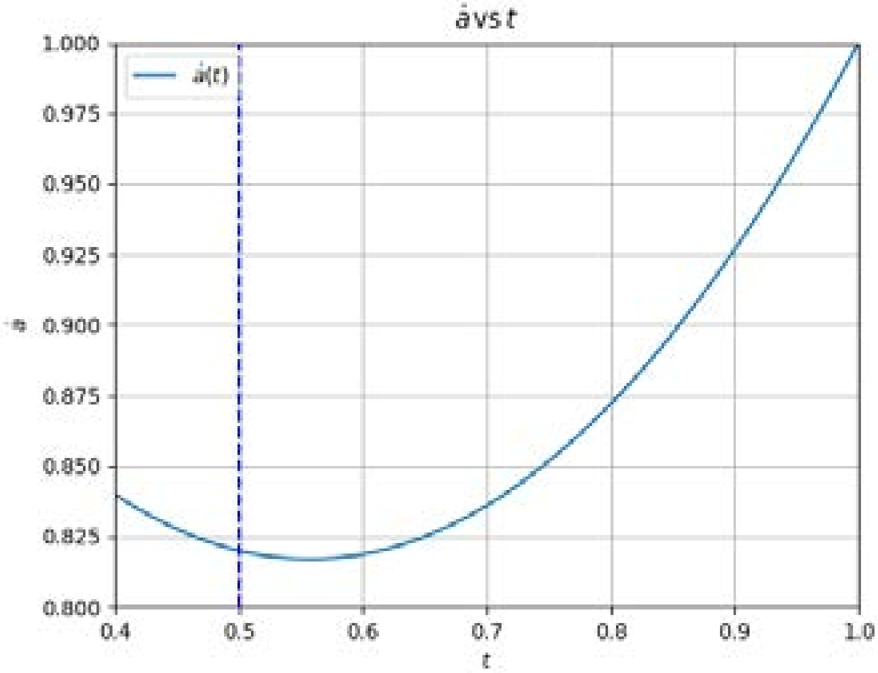

Figure 2. Highlights $\dot{a}(t)$, showing a slight deviation near specific intervals, indicating transition phases.

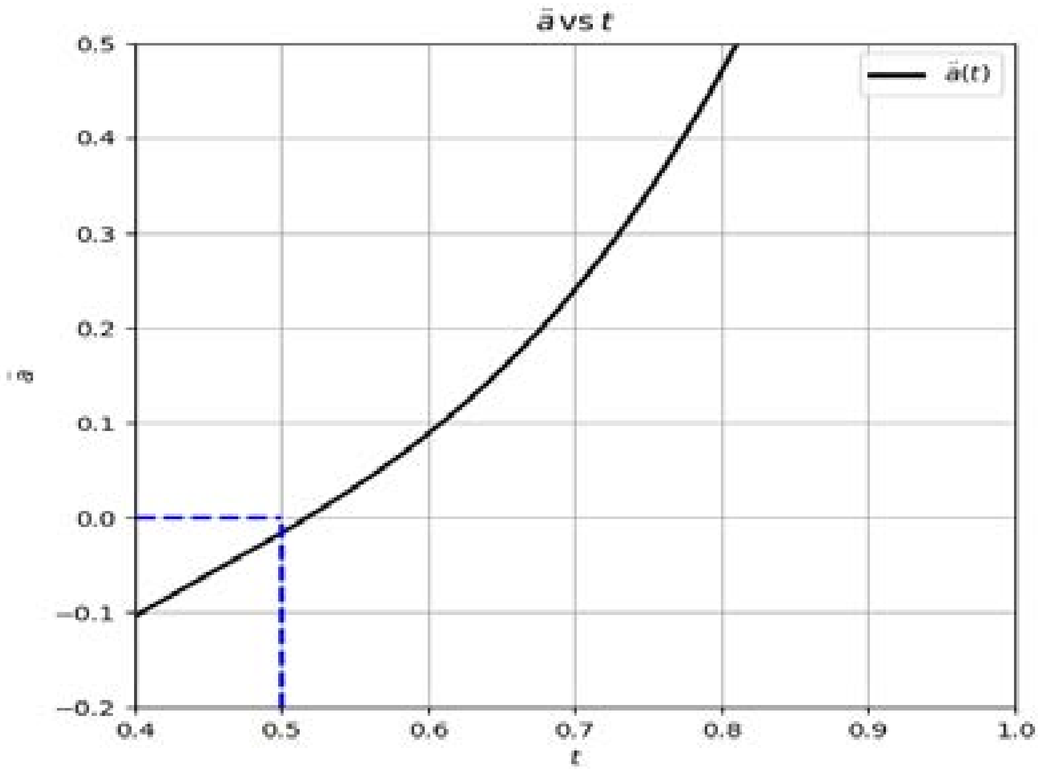

Figure 3. showcases $\ddot{a}(t)$, with the negative values transitioning to positive as the acceleration overtakes deceleration.

Graphical Analysis of $a(t)\dot{a}(t)$ and $\ddot{a}(t)$ Figures 1, 2, and 3 present the behavior of the scale factor a(t), its first derivative $\dot{a}(t)$, and second derivative $\ddot{a}(t)$, respectively, over time.

Key Observations. From these graphs, it becomes evident that the scale factor exhibits a steady increase, while the acceleration $\ddot{a}(t)$ undergoes significant variation due to the effects of generalized gravity.

Figure 1 *a (t)* increases linearly $a(t) \propto t^n\ n > 1$ and $a(t) \propto e^{Ht}$. Figure 2 *a (t)* – changes slightly. The variable acceleration model $a(t) \propto sinh^{\frac{2}{3}}(\alpha t)$ and $a(t) \propto t^n + \gamma e^{\lambda t}$, as in the transition from matter to dark energy. Figure 3 *a (t)* from negative to positive value.

The physical meaning $\dot{a}(t).\dot{a}(t) = \frac{da}{dt}$. This is the rate of change in the scale of space over time, that is, how much $\dot{a}$ *(t > 0)* – The universe is expanding, and $\dot{a}$ *(t < 0)* compression occurs (in a collapsing model), $\dot{a}$ *(t)* the bigger, the faster the expansion takes place.

The physical meaning $\ddot{a}(t).\ddot{a}(t) = \frac{d^2a}{dt^2}$. This is an acceleration of the expansion or deceleration of the universe $\ddot{a}$ *(t > 0)* – the universe is expanding at an accelerating rate, $\ddot{a}$ *(t < 0)* – expansion slows down. $\ddot{a}$ *(t) = 0* the scale factor is uniform. This equation is included in the second-order equation $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$.

Conclusion

The derived formulas and numerical results provide a comprehensive view of the relationship between the scale factor and redshift in generalized gravity models. This study emphasizes the utility of alternative approaches to describing the universe's dynamics. By considering extensions of classical general relativity, we gain deeper insights into the accelerating expansion of the universe.

The contribution of the authors
Yerzhanov K.K. – set tasks and general adjustments;
Sergazina A.M. – writing an article, collecting material;
Murzakul T.R. – plotting graphs;
Baurzhan G.B. – plotting graphs.

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Жалпыланған гравитациялық модельдердегі гравитациялық қызыл ауысу

Аңдатпа. Бұл мақалада Эйнштейннің жалпы салыстырмалылық теориясының шеңберінен шығатын кеңейтілген гравитациялық модельдер шеңберіндегі гравитациялық қызыл ығысу құбылысы зерттелген. Гравитациялық қызыл ығысу кеңістік-уақыт қисықтығының негізі байқылатың көрінісі болып табылады және гравитациялық теорияларды сынау үшін маңызды құрал ретінде қызмет етеді. Жалпы салыстырмалылық шеңберінде ол гравитациялық өрістегі уақыттың кеңеюінің салдары ретінде түсіндіріледі. Бұл жұмыс әртүрлі жалпыланған гравитациялық қызыл ығысу көрінісінің ерекшеліктерін қарасырады және бұл модельдердің болжамдарын бақылау нәтижелерімен, атап айтқанда, жинақы объектілердің жанында және космологиялық масштабта салыстырмалы талдау жүргізеді. Біз хаббл параметрі арқылы байланыстыра отырып, қызыл ығысу мен ғаламның нормаланған масштаб факторы арасында байланыс орнатамыз. Біз redshift уақыт эволюциясын зерттеу үшін аналитикалық және сандық тәсілдерді әзірлейміз. Басқару теңдеулерін шеше отырып, өрнектерді аламыз үшін, және олардың туындылары уақытқа қатысты. Нәтижелер қазіргі космологиялық бақылауларға сәйкес келеді және ғаламның кеңею динамикасы туралы жаңа түсінік береді. Сонымен қатар, уақыт бойынша масштаб факторының эволюциясын, оның бірінші және екінші туындыларын визуализациялау ұсынылған. Бұл нәтижелер гравитациялық күштер мен ғарыштық кеңею арасындағы күрделі өзара әрекеттесуді жақсырақ түсінуге ықпал етеді

Түйін сөздер: Гравитациялық қызыл ығысу; масштаб коэффициенті; Хаббл параметрі; космологиялық модельдер; жалпыланған ауырлық күші.

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Гравитационное красное смещение в обобщенных гравитационных моделях

Аннотация. В данной статье исследуется явление гравитационного красного смещения в рамках расширенных гравитационных моделей, которые выходят за рамки традиционной общей теории относительности Эйнштейна. Гравитационное красное смещение представляет собой ключевое наблюдаемое проявление кривизны пространства-времени и служит важным инструментом для проверки теорий гравитации. В рамках общей теории относительности (ОТО) оно объясняется как следствие замедления времени в гравитационном поле. В этой работе рассматриваются особенности проявления гравитационного красного смещения в обобщенных гравитационных теориях, проводится сравнительный анализ предсказаний этих моделей с результатами наблюдений в частности, вблизи компактных объектов и на космологических масштабах. Мы устанавливаем взаимосвязь между красным смещением и нормализованным масштабным коэффициентом Вселенной, связывая его через параметр Хаббла. Мы разрабатываем как аналитические, так и численные подходы для изучения изменения красного смещения во времени. Решая основные уравнения, мы получаем выражения для и их производные по времени. Результаты согласуются с текущими космологическими наблюдениями и дают новое представление о динамике расширения Вселенной. Кроме того, представлены визуализации изменения масштабного коэффициента, его первой и второй производных с течением времени. Эти результаты способствуют лучшему пониманию сложного взаимодействия между гравитационными силами и космическим расширением.

Ключевые слова: гравитационное красное смещение; масштабный коэффициент; параметр Хаббла; космологические модели; обобщенная гравитация.

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