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Hydrodynamic Equations and Ionization Waves in Fissioning Plasmas

Abstract: the study defines coupled self-consistent Boltzmann equations for fission fragments and created primary electrons are defined for weakly ionized dense plasma irradiated by fission fragments. There has been studied the kinetics of the energy formation of fast particles in a plasma based on the equations. The authors have found and analysed steady-state analytical solutions for fission fragments and the primary electrons' functions of energy distribution for the helium-3 plasma irradiated by neutron flux. The results of the study have been compared with Monte Carlo energy spectra calculations.

Keywords: Boltzmann equation, fission fragment, fissioning plazma, ionization wave, hydrodynamic equations.

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Introduction. Any gas irradiated by high-energy particles [1-2] demonstrates its modified chemical properties which might be applied in various technologies, such as the direct transformation of nuclear energy into coherent optical radiation [1],[3]. If the test gas under study contains a fissionable component interacting with neutrons, such as helium-3 [^3He] or uranium hexafluoride [UF_6], and sets into the thermal neutron flux, then the created plasma turns into a unique physical object. Within this physical condition, it is possible to realize the direct transformation of the nuclear energy into electromagnetic radiation of excited atoms.This challenging technology has some internal unsolved problems awaiting solution, and the major is the energy distribution of fission fragments of primary electrons. The evolution of these energy spectra to its stationary state is completely different from the electric discharge. The study of kinetic processes in various modern publications based on the solution of the Boltzmann equation is focused and applied to a wide range of physical phenomena, from the description of a discharge in external electric fields to the pumping of gas mixtures by electron beams. Plasma induced by fission fragments also might be described by the Boltzmann equation, which, certainly, should be definitely adjusted to the physical conditions for gaseous mixtures with fissioning components.

Methods. Methods have been developed for the complete analysis and solution of the Boltzmann equation for inhomogeneous gases.

Full extent analysis and solution methods of Boltzmann equation [4] applied for non-uniform gases were developed in [3] . In one of the earliest works Boltzmann equation was applied to a weakly ionized plasma in the periodical electric field. In this research the high frequency gas discharge was studied for a slightly ionized plasma within the following limiting conditions. It was assumed that electrons, ions and excited molecules densities supposed to be small compared to the density of buffer gas [BG] molecules; elastic collisions prevailed over the inelastic ones. Test gas regarded at densities for which the hydrodynamic approach is reasonable, and the frequency of external electric field assumed to be higher than the own plasma frequency. The major of part of is the adjustment of Boltzmann equation for a plasma electron distribution influenced by external electric field of high frequency. The assumption that initial and final velocities are close to each other therefore led to sufficient limitations for further applications. It should be also mentioned that Boltzmann kinetic equation under the same assumptions was applied to fissioning plasma , but the key problem mentioned above that how the primary electrons spectra will look like was not even discussed.

The presence of electron density gradient implies that the external field should be self-consistent. Secondary electrons spectra are the subject of Townsend ionization processes , which should be also included in Boltzmann kinetic equation. Electron beam impact taken as an ionization source was studied in [5] and Boltzmann equation was also used without any transformation as in [6]. The appearance of, so called, runaway electrons in a fully ionized plasma, affected by sufficiently strong electric field, was analyzed in . Following to [8], the most of the electrons on their mean free path receive more energy from electric field than they lose through the inelastic collisions, and that is why the electrons are to be continuously accelerated. The steady state Boltzmann equation was presented with the external force which was treated as bremsstrahlung. Certainly, this case also might be considered if we have some information about what is the energy band where this braking force demonstrates its appearance and its absence. The identification of fast electrons with energy around MeV (runaway electrons) was not discussed or presented and this problem might be solved in statistical approach [9]. It should be pointed out that the energy band of the electron degradation spectra connected with its radiation (bremsstrahlung)was not defined but included in the Boltzmann equation. It turned out to be that the X-rays, might be treated like electrons due to their Broglie wavelength and experimental pronounced resolution of runaway electrons' appearance left to be an unsolved problem. Following to, we take the interactions between the test or incident particle with atoms as those which should not be treated like two rigid spheres, but rather like interactions of incident particle with a complex dynamical system of charged particles moving in the state of dynamical equilibrium.

Result. 1. Boltzmann kinetic equations for fission fragments and primary electrons.

Let us denote fission fragments species by and electrons by and assume that their concentration much less than the concentration of neutrals or buffer gas . As a result of this assumption we neglect with fission fragments and electrons interactions between each other. We also denote functions of energy distribution of fission fragments and electrons as follows

Then the j -type of fission fragments function of energy distribution is described by Boltzmann equation as follows:

$$\begin{gathered}\partial_\mu \tilde{v}_j(t,\vec{r},\vec{\xi}) = \Omega_j^{fission}(t,\vec{r},\vec{\xi}) + S_j^{ion}(\vec{\xi_j'} \to \vec{\xi}_j, \tilde{v}_j) \\ S^{el}(\vec{\xi_j'} \to \vec{\xi}_j, \tilde{v}_j) - \mathfrak{L}_j^{rec}(\vec{\xi_j} \to \vec{\xi_j'}), \tilde{v}_j * \tilde{v}_e\end{gathered} \tag{1}$$

We underline that all terms a weakly ionized plasma are linear with respect to the function $\mathfrak{F}_j(t,r,\xi)$ except the recombination term which includes the function of electron distribution and presents non linear term $\mathfrak{F}_j * \mathfrak{F}_e$.

Here, the $\Omega_j^{fission}$ is the source of j-type of fission fragments due to nuclear fission of any fissioning component density $\{\, FC \to {}^3He, {}^{235}UF_{6,}{}^7Be, {}^{10}B\,\}$ in neutron flux and equals as follows:

$$\begin{gathered}\Omega_j^{fission}(n + FC \to \sum_k \mathfrak{F}^k(\xi_o)) = \\ = n_{FC} \cdot \Phi(\mathrm{t}, \overrightarrow{r})\sigma^{fission}(\varepsilon_\mathrm{n}) * \delta(\overrightarrow{\xi}_j - \overrightarrow{\xi}_j^0)\end{gathered} \tag{2}$$

In this equation n_{FC} is fissioning component concentration, like ${}^3He, {}^{235}UF_6$.

$\Phi(t, \overrightarrow{r})$ — a neutron flux which is a function of space and time coordinates. Neutron flux depends on the definite active zone type in test reactor, $\sigma^{fission}(\varepsilon_n)$ is a nuclear reaction, like followings:

$$\begin{gathered}{}^3He + n \to p + T \ + 0.76\ Mev, \\ {}^{235}UF_6 + n \to ff_l + ff_h + 165 Mev\end{gathered}$$

cross section,

$\delta(\overrightarrow{\xi}_j - \overrightarrow{\xi}_j^o)$ -j-type fission fragment energy distribution at the point of its initial energy presented by delta function. The second term is equal to:

$$S_j^{ion}(\xi_j) = n_{FC} \int_{\xi_j+I}^{\xi^0} \delta(\xi_{j+I} - G^{ion}(\xi_j^{'}, \xi_j))$$
$$\int_I^{E_j^0-I} \left(\Im_j(\xi_j^{'}) - \mathfrak{F}(\xi_j)\right) \xi_j^{'} \Omega^{ion}(\xi_j^{'}, \triangle E_j) p_j^{ion}(\xi) d(\triangle E_j) d\xi_j^{'} \quad (3)$$

In this equation n_{FC} is a buffer gas concentration (in our case it is the fissioning component concentration), I -is buffer gas ionization potential, $\triangle E_j$ is j-fragment's energy loss, which is equal to the sum of ionization potential and an energy acquired by released electron. Differential cross section and probability of the ionization are [10]:

$$\Omega_j^{ion}(\xi_j, I) = \frac{6.5610^{-14} Z_j^2}{(I)^2 E_j} \left(\frac{\xi_e^{av}}{\xi_j}\right)^2 \left[\frac{\xi_j^2}{\xi_j^2 + (\xi_e^{av})^2}\right]^{\frac{3}{2}} *$$
$$\left[\frac{\xi_j^2}{\xi_j^2 + (\xi_e^{av})^2} + \frac{4}{3} ln\left(2.7 + \frac{\xi_j}{\xi_e^{av}}\right)\right] * \left(1 - \frac{I}{\Delta E_{max}}\right)^{1+\frac{\xi_j^2}{(\xi_e^{av})^2}}, \quad (4)$$
$$\Delta E_{max} = 4E_e^{av} (\frac{\xi_j}{\xi_e^{av}})^2 \left(1 + \frac{\xi_e^{av}}{\xi_j}\right)$$

and

$$p_j^{ion}(\xi) = \frac{\Omega_j^{ion}(\xi)}{\sum\limits_{k=1}^{m} \Omega_j^k(\xi)} \quad (5)$$

Following to [10], [11] we define delta-function $\delta(\xi_j - G(\xi_j^{'}, \xi_j))$ as a function, which returns back value of j-type fragment from another phase volume ($\xi_j^{'} \to \xi_j$) taking into the consideration definite interaction potential. $G(\xi_j^{'}, \xi_j)$ is collision integral of the following type :

$$\Omega^{ion} = 2\pi b(\chi) \left(\frac{db(\chi)}{d\chi}\right) d\chi,$$
$$\phi^l = \int_{b_{min}}^{\infty} (1 - \cos^l \chi] gbdb, \quad (6)$$

here χ is deflection angle.
Correspondent leakage term is presented as:

$$\mathfrak{L}_j^{ion}(\xi_j) = n_{FC} \int_\omega \Im_j(\xi_j) \xi_j \Omega_j^{ion}(\xi_j) p_j^{ion}(\xi_j) d\omega \quad (7)$$

here ω is a spherical angle. Excitation terms are equal as follows:

$$S_j^{exc}(\xi_j) = \sum\nolimits_{k=1}^{k=m} p_{jk}^{exc}(E_j) n_{FC} * \int_{\xi_j+I_k}^{\xi^0} \delta(\xi_j - G_k^{exc}(\xi_j^{'}, \xi_j))$$
$$* \int_{I_l}^{E_j^0-I_k} (\Im_j(\xi_j^{'}) - \Im_j(\xi)) \xi_j^{'} \Omega_{jk}^{exc}(\xi_j^{'}, \Delta E_{j,k}) d(\Delta E_{j,k}) d\xi_j^{'} \quad (8)$$

Differential cross section of excitation of k-level and its probability are equal:

$$\Omega_{j,k}^{exc}(\xi_j, I_k) = \frac{6.5610^{-14} Z_j^2}{(I(k))^2 E_j} \left(\frac{\xi_e}{\xi_j}\right)^2 \left[\frac{\xi_j^2}{\xi_j^2 + (\xi_e^{av})^2}\right]^{\frac{3}{2}}$$
$$\left[\frac{\xi_j^2}{\xi_j^2 + (\xi_e^{av})^2} + \frac{4}{3} ln\left(2.7 + \frac{\xi_j}{\xi_e^{av}}\right)\right]\left(1 - \frac{I_k}{\Delta E_{max}}\right)^{1+\frac{\xi_j^2}{(\xi_e^{av})^2}} \tag{9}$$

The probability of k-level excitation is equal:

$$p_j^k(\xi) = \frac{\Omega_j^k(\xi)}{\sum\limits_{k=1}^{n} \Omega_j^k(\xi)} \tag{10}$$

Correspondent leakage terms for excitation term are presented as:

$$\mathfrak{L}_j^{exc}(\xi_j) = n_{FC} \sum_k^M \int_\omega \mathfrak{I}_j(\xi_j)\xi_j \Omega_{j,k}^{exc}(\xi_j) p_{j,k}^{exc}(\xi_j) d\omega \tag{11}$$

It should be pointed out that the loss of energy in the elastic collisions might be evaluated as follows [12] :

$$\Delta E^{el} = 2\frac{m_j}{M} E_j (1 - \cos\chi) \tag{12}$$

here m_j, M mass of fission fragment and colliding atom's mass, $\triangle E, E_j$ energy loss and fission fragment initial energy. The terms in the Equation $\sim$ (1) are presented as follows:

$$S_j^{el}(\xi_j) = n_{FC} \int_{\xi_j + \triangle E^{el}}^{\xi^0} \delta(\xi_j - G^{el}(\xi_j^{'}, \xi_j))$$
$$\int_0^{E_j^0 - \triangle E^{el}} \left(\mathfrak{I}_j(\xi_j^{'}) - \mathfrak{F}_j(\xi)\right) \xi_j^{'} \Omega_j^{el}(\xi_j^{'}, \triangle E_j) p_j^{el}(\xi_j) d(\triangle E_j) d\xi_j^{'} \tag{13}$$

The leakage term connected with elastic collisions is presented as follows:

$$\mathfrak{L}^{el}(\xi_j) = n_{FC} \int_\omega \mathfrak{I}_j(\xi_j)\xi_j \Omega_j^{el}(\xi_j, \chi) p_j^{el}(\chi^{av}(\xi_j)) d\omega \tag{14}$$

The recombination term strongly connected with electrons energy distribution function and equals:

$$\mathfrak{L}_j^{rec} = \int_0^{\xi_{emax}} \Omega_{j,e}^{rec}(\xi_j, \xi_e) \mathfrak{F}_j(t, r, \xi_j) *$$
$$* \mathfrak{F}_e(t, r, \xi_e) p_j^{rec}(\xi_j - \xi_e) d\xi_e \tag{15}$$

Energy distribution function for electrons is defined by the following equation:

$$\begin{gathered}\partial_\mu \mathfrak{F}_e(t,\vec{r},\vec{\xi_e}) = \sum\nolimits_j \left[S_{pe}^{ion}(\mathfrak{F}_j \to \mathfrak{F}_e)\right] + \\ +S_e^{ion}(\xi_e^{'} \to \xi_e, \mathfrak{F}_e) - \mathfrak{T}_e^{ion}(\xi_e \to \xi_e^{'}, \mathfrak{F}_e) + \\ +\sum\nolimits_k^m S_e^{exc}(\xi_e^{'} \to \xi_e, \mathfrak{F}_e) - \mathfrak{T}_e^{exc}(\xi_e \to \xi_e^{'} \mathfrak{F}_e) + \\ +S_e^{el}(\xi_e^{'} \to \xi_e, \mathfrak{F}_e) - \mathfrak{T}_e^{el}(\xi_e \to \xi_e^{'}, \mathfrak{F}_e) + \\ +\sum\nolimits_j^m \mathfrak{L}_e^{rec}(\xi_e \to \xi_e^{'}, (\mathfrak{F}_e, \mathfrak{F}_j))\end{gathered} \tag{16}$$

The first term of Equation $\sim$ (16) presents primary electrons' formation from all types of fission fragments and equals to:

$$\begin{gathered}S_{pe}^{ion} = p_e^{ion}(E_e) n_{FC} \int_{v_e^{min}}^{v_e} \delta(v_e - G(v_j^{'}, v_j) dv_j^{'} \\ \int_I^{E_0 - I} \mathfrak{J}_j(v_j^{'}) v_j^{'} \Omega_j^{ion}(v_j^{'}, \Delta E_j) d(\Delta E_j)\end{gathered} \tag{17}$$

The second term describes the flow of electrons from outer phase volume due their ability to create secondary ones (electrons born by primary electrons, so called, secondary generation):

$$\begin{gathered}S_e^{ion} = p_e^{ion}(E_e)[BG] \int_{v_e^{min}}^{v_e^0} \delta(v_e - G(v_e^{'}, v_e) dv_e^{'} \\ \int_I^{E_e - I} \mathfrak{J}_e(v_e^{'}) v_e^{'} \Omega_e^{ion}(v_e^{'}, \Delta E_e) d(\Delta E_e)\end{gathered} \tag{18}$$

where

$$\begin{gathered}\Omega_e^{ion}(v_e, \triangle E) = \frac{Z_{\mathfrak{J}}^2}{(\triangle E_e)^2 E_e} \left(\frac{\overline{v_e}}{v_e}\right)^2 \left[\frac{v_e^2}{v_e^2 + \overline{v_e^2}}\right]^{\frac{3}{2}} \\ \left\{\left[\frac{v_e^2}{v_e^2 + \overline{v_e^2}}\right] \frac{\triangle E}{I} + \frac{4}{3} ln \left(2.7 + \left(\frac{v_e}{\overline{v_e}}\right)^{\frac{1}{2}}\right)\right\} \\ (1 - \frac{\triangle E}{\triangle E_{max}})^{(1 + \frac{v_e^2}{v^2{}_e})}\end{gathered} \tag{19}$$

$$\mathfrak{L}_e^{ion}(\xi_j) = n_{FC} \int_\omega \mathfrak{J}_e(\xi_e) \xi_e \Omega_e^{ion}(\xi_e) p_e^{ion}(\xi_e) d\omega \tag{20}$$

here ω is a spherical angle.

Excitation terms are equal as follows:

$$S_e^{exc}(\xi_j) = \sum\nolimits_{k=1}^{k=m} p_{ek}^{exc}(E_e) n_{FC} *$$
$$* \int_{\xi_e+I_k}^{\xi^0} \delta(\xi_e - G(\xi_e^{'}, \xi_e)) * \tag{21}$$
$$* \int_{I_k}^{E_e^0 - I_k} \mathfrak{J}_e(\xi_e^{'}) \xi_e^{'} \Omega_{ek}^{exc}(\xi_e^{'}, \triangle E_j) d(\triangle E_e) d\xi_e^{'}$$

Differential cross section of excitation of k-level and its probability are equal:

$$\Omega_{e,k}^{exc}(\xi_j, I_k) = \frac{6.5610^{-14}}{(I(k))^2 E_e} \left[\frac{\xi_e^2}{\xi_e^2 + (\xi_e^{av})^2}\right]^{\frac{3}{2}} *$$
$$* \left[\frac{\xi_e^2}{\xi_e^2 + (\xi_e^{av})^2} + \frac{4}{3} \ln\left(2.7 + \frac{\xi_e}{\xi_e^{av}}\right)\right] * \tag{22}$$
$$* \left(1 - \frac{I_k}{\triangle E_{e,max}}\right)^{1 + \frac{\xi_e^2}{(\xi_e^{av})^2}}$$

The probability of k-level excitation is equal:

$$p_e^k(\xi) = \frac{\Omega_e^k(\xi)}{\sum\limits_{k=1}^{n} \Omega_e^k(\xi)} \tag{23}$$

Correspondent leakage terms for excitation term are presented as:

$$\mathfrak{L}_e^{exc}(\xi_j) = n_{FC} \sum\nolimits_k^M \int_\omega \mathfrak{J}_e(\xi_e) \xi_e \Omega_{e,k}^{exc}(\xi_e) p_{e,k}^{exc}(\xi_e) d\omega \tag{24}$$

It should be pointed out that the loss of energy in the elastic collisions might be evaluated as follows [[**?**]]:

$$\Delta E^{el} = 2 \frac{m_e}{M} E_e (1 - \cos\chi) \tag{25}$$

here m_j, M mass of fission fragment and colliding atom's mass, $\triangle E, E_e$ energy loss and electron's initial energy. The elastic collision terms in the Equation $\sim$ (16) are presented as follows:

$$S_e^{el}(\xi_j) = n_{FC} \int_{\xi_e + \triangle E^{el}}^{\xi^0} \delta(\xi_e - G(\xi_e^{'}, \xi_e))$$
$$\int_0^{E_e^0 - \triangle E^{el}} \mathfrak{J}_e(\xi_e^{'}) \xi_e^{'} \Omega_e^{el}(\xi_e^{'}, \triangle E_e) p_e^{el}(\xi_e) d(\triangle E_e) d\xi_e^{'} \tag{26}$$

The leakage term connected with elastic collisions is presented as follows:

$$\mathfrak{L}^{el}(\xi_e) = n_{FC} \int_\omega \mathfrak{J}_e(\xi_e) \xi_e \Omega_e^{el}(\xi_e, \chi) p_e^{el}(\chi^{av}(\xi_e)) d\omega \tag{27}$$

The recombination term strongly connected with electrons energy distribution function and equals:

$$\begin{gathered}\mathfrak{L}_e^{rec} = \sum\nolimits_j \int_0^{\xi_{emax}} \Omega_{e,j}^{rec}(\xi_j, \xi_e) \mathfrak{F}_j(t, r, \xi_j) * \\ * \mathfrak{F}_e(t, r, \xi_e) p_e^{rec}(\xi_j - \xi_e) d\xi_e\end{gathered} \tag{28}$$

2. Hydrodynamic equations of fission fragments and electrons.
We rewrite hydrodynamical equations for fission fragments and electrons in full presentation:

$$\frac{\partial n_j}{\partial t} + \nabla \overrightarrow{j_j} = S_j^{fission} - k_j^{ion} n_j - k_j^{chex} n_j - k_j^{el} n_j \tag{29}$$

$$\frac{\partial}{\partial t}\left\{\frac{3}{2} n_j T_j\right\} = S_j E_j^0 - k_{j,\varepsilon}^{el}(T_j - T_0) - k_\varepsilon^{chex} n_j \tag{30}$$

$$\frac{\partial n^+}{\partial t} = k_j^{chex} n_j - k^{rec} n^+ n_e \tag{31}$$

$$\begin{gathered}\frac{\partial n_e(t,\vec{r})}{\partial t} + \nabla \vec{j}_e(t,\vec{r}) = S_e^{pe}(t,\vec{r}) + k^T(t,\vec{r}) n_e(t,\vec{r}) - \\ - k^{aff}(t,\vec{r}) n_e(t,\vec{r}) - k^{rec}(t,\vec{r}) n_e^2(t,\vec{r})\end{gathered} \tag{32}$$

$$\frac{\partial}{\partial t}\left\{\frac{3}{2} n_e k T_e\right\} = \sum_j S_j^{pe} E_e^{j0} - \chi\{T_e - T_0\} - \lambda_e \nabla Te - k_\varepsilon^{aff} n_e - k_\varepsilon^{rec} n_e \tag{33}$$

$$\vec{j}_e = -D_e \nabla n_e + b_e n_e \vec{E} - D_e^T \nabla T_e \tag{34}$$

$$\nabla \vec{E} = -4\pi(n_e + n^- - n^+) \tag{35}$$

One should be aware, that variables t and $\overrightarrow{r}$ are hydro dynamical ones, but $\overrightarrow{\xi}$ is microscopic variable and it should be treated as a particle's (fission fragments or electrons) velocity. We assume that buffer gas is not moving and weak electric field is applied. Then we have the presence of following currents:

$$\begin{gathered}\overrightarrow{J_{\mathfrak{J}}} = D_j grad\, n_j + \mu_j grad\, \varphi + D_j^T grad\, T_j, \\ \overrightarrow{J_e} = D_e grad\, n_e + \mu_e grad\, \varphi + D_e^T grad\, T_e\end{gathered} \tag{36}$$

The continuity equations, equation of fission fragments and electron's motion, and energy transport are presented as follows:

$$\begin{gathered}\frac{\partial n_j}{\partial t} + div\overrightarrow{J}_{\mathfrak{J}} = S(\mathfrak{J}) - \mathfrak{L}(\mathfrak{J}),\\ \frac{\partial n_e}{\partial t} + div\overrightarrow{J_e} = S_e - \mathfrak{L}_e,\\ \triangle\varphi = 4\pi e\left(\sum\nolimits_j n_j - n_e\right)\end{gathered} \tag{37}$$

In Equation (37) the electric current in the weak external electric field equals:

$$\begin{gathered}\vec{J} = \sum\nolimits_j \vec{J}_{\mathfrak{J}(j)} + \vec{J}_e =\\ = \sum\nolimits_j eZ_{\mathfrak{J}(j)} n_{\mathfrak{J}(j)} \left\langle \overrightarrow{v}_{\mathfrak{J}(j)} \right\rangle + en_e \left\langle \overrightarrow{v}_e \right\rangle,\\ \left\langle \overrightarrow{v}_{\mathfrak{J}(j)} \right\rangle = b_{\mathfrak{J}(j)} grad\,\varphi,\ \ \left\langle \overrightarrow{v}_e \right\rangle = b_e grad\,\varphi\end{gathered} \tag{38}$$

Electrons diffusion coefficient and electric mobility are as follows:

$$\begin{gathered}D_e = n_{FC}\int_o^{E_{max}} \Omega_e^{el}(\triangle E, \xi)\{f_e^{(0)} + \varepsilon f_e^{(1)} + ..\}\xi d(\triangle E) d\xi\\ b_e = \frac{em_e}{kT_e}\int_o^{E_{max}} \Omega_e^{el}(\triangle E, \xi)\{f_e^{(0)} + \varepsilon f_e^{(1)} + ..\}\xi^2 d(\triangle E) d\xi\end{gathered} \tag{39}$$

3. Ionization waves in fissioning plasma.

The phenomena of breakdown in gaseous systems described in [13], [14], [15] emphasize,that streamers formation is initiated by ionization waves on the tips of electron avalanche and result the breakdown in pre ionized gases influenced by external electric field.The theoretical study of ionization waves was based on the following system of hydrodynamical equations [16]:

$$\begin{gathered}\frac{\partial n_e}{\partial t} + \nabla n_e \vec{v}_e = \alpha(E) n_e - \beta n_e n_i\\ \frac{\partial n_i}{\partial t} = \alpha(E) n_e - \beta n_e n_i\\ \nabla\vec{E} = \varepsilon_0^{-1} e(n_e - n_i)\\ \vec{v}_e = \mu_e \vec{E} - D\frac{\nabla n_e}{n_e}\end{gathered} \tag{40}$$

Here β is the coefficient of dissociative recombination,and μ_e is the electron mobility.Following to the existing point of view,the avalanche -streamer transition takes place at the moment of time t_{cr} when the resulting electric field $\overrightarrow{E} = \overrightarrow{E}_0 + \overrightarrow{E}'$ tends to zero at the point $z = z_{cr}$ along the avalanche axis. Introducing parameter χ , which is equal $\chi(E_0) = \frac{\alpha}{v_e}$,the critical value z_{cr} obtained as follows [17], [14]:

$$\chi(E_0) z_{cr} = 17.9 + Ln(\varepsilon z_{cr}) \tag{41}$$

However,the value ε is the key problem which is not yet solved.And main objection which should be added is the time mean electrons energy is the function of time and it is possible to solve this problem is utilize the Boltzmann equation,and for the case of fissioning plasma equations((2) and (6)). The relaxation time scale of fission fragments is much less than for electrons within which fission fragment's energy equalizes with the thermal energy of buffer gas. The buffer gas heated by fission fragments not only due to the elastic collision processes , but by inelastic collisions ,including ionization(recoil), excitation(recoil). The expansion small parameter might be chosen to be equal to Knudsen number ($\varepsilon = Kn$), or Mach number ($\varepsilon = M$) in the case when buffer gas is moving [18]. However, the Knudsen number as well as Mach number does not reflect the great difference in the rates of relaxation of fission fragments and electrons in their degradation to equilibrium states. The presence of stationary source of ionization due to fissioning processes shifts the mean value of electrons to higher energy values. In fissioning plasma we also may identify mean value of fission fragment energy E_j^{av} ,mean value of electrons energy E_e^{av} , parameter $\delta = \frac{m_e}{M}$,mean free flight length $\lambda_{j,e} = \frac{1}{\sum\limits_{j,e} \sigma_{j,e} n_{buffer\ gas}}$. The Maxwellian time for heavy fission fragments is around $10^{-11}s$ and for electrons $10^{-8} - 10^{-6}s$ [19], [20]. To combine all these factors working together we do the re normalization in time scale in energy spectra evolution of heavy particles and electrons and chose the expansion parameter as follows [21]

$$\varepsilon_{j,e} = \delta * \sqrt{\frac{E_{j,e}^{Maxwell}}{E_{j,e}^{Max}}}, \quad \delta = \frac{m_{j,e}}{M} \tag{42}$$

Time relaxation of heavy fragments undergoes in two or three orders quicker than electrons relaxation time and in the expansion solutions the functions of fission fragments and electrons might be presented as follows:

$$\begin{gathered} f_j(t, \vec{r}, \vec{\xi}) = f_j^{(0)}(t, \vec{r}, \vec{\xi}) + \varepsilon_j f_j^{(1)}(t, \vec{r}, \vec{\xi}) + \varepsilon_j^2 f_j^{(2)}(t, \vec{r}, \vec{\xi})..., \\ f_e(t, \vec{r}, \vec{\xi}) = f_e^{(0)}(t, \vec{r}, \vec{\xi}) + \varepsilon_e f_e^{(1)}(t, \vec{r}, \vec{\xi}) + \varepsilon_e^2 f_e^{(2)}(t, \vec{r}, \vec{\xi})... \end{gathered} \tag{43}$$

The time relation rate are subdivided by the prompt ones corresponding to zero approximation, then the second time scale is regarded as the fist one divided by ε and the more slow rate divided by ε^2 and so on. Utilizing the way of splitting the two basic equations we come to the following series of equations which might be solved as a time dependent functions. The zero approximation of the Equation gives the following equation:

$$\begin{gathered} \frac{\partial f_j^{(0)}(t, \vec{r}, \vec{\xi})}{\partial t} = S_j^{ff} - \left\{ n_{FC} \int_I^{E_0^j - I} \delta(\xi - \xi_0)) \xi_0 \Omega^{ion}(\xi_0, \triangle E) * p_j^{ion}(\xi_0) d(\triangle E \right\} \\ * \left(f_j^{(0)}(t, \vec{r}, \vec{\xi}) \right) \end{gathered} \tag{44}$$

The first term in Equation (45) presents the rate of fission fragments formation and is equal to:

$$\begin{gathered} S_j^{ff} = n_{FC}(t, \vec{r}) * \Phi(t, \vec{r}) * \sigma_{fission}\, \delta(\xi - \xi_j^0) \ , \\ \int_0^\infty \delta(\vec{\xi} - \vec{\xi}_j^0) \, \mathrm{d}\, \vec{\xi} = 1 \end{gathered} \tag{45}$$

here $n_{FC}(t, \vec{r})$ is fissioning component concentration, like ^3He , $^{235}UF_6$, $\Phi(t, \vec{r})$ is a neutron flux, which is a function of time and space coordinates, I ionization potential. Then the zero approximation of the function of energy distribution of j-type fission fragments at the point of initial kinetic energy might $E_j^0 = \frac{m_j (\xi_j^o)^2}{2}$ might be taken as a delta function $\delta(\vec{\xi}_j - \vec{\xi}_j^o)$.

Neutron flux depends on a definite active zone type in a test reactor, $\sigma_{fission}(\varepsilon_n)$ is the cross section of neutrons with fissioning components resulting fission fragments. Any fissioning nuclear reaction runs like followings:

$$
\begin{gathered}
{}^3He + n \to p + T + 0.76MeV, \\
{}^{235}UF_6 + n \to ff_l + ff_h + 165MeV.
\end{gathered} \tag{46}
$$

The first approximation lasts in the scale $\frac{pico\ sec}{\varepsilon}$ and defined as follows:

$$
\begin{gathered}
\frac{\partial_j^{(1)}\vec{r}\vec{\xi}}{\partial t} = -\{n_{FC}\int_I^{E_0^j - I}\left(f_j^{(0)}(t,\overrightarrow{r},\xi^{'})\right)\delta(\xi^{'} - G(\xi,\xi^{'}))\xi\Omega^{ion}(\xi^{'},\Delta E) \\
*p_j^{ion}(\xi^{'})d(\Delta E)d\xi^{'}\} - \\
-\sum\nolimits_k\{n_{FC}\int_{U_k}^{E_0^j - U_k}(f_j^{(0)}(t,\vec{r},\vec{\xi^{'}}))\delta(\xi^{'} - G(\xi,\xi^{'}))\xi\Omega_j^{exc}(\xi^{'},\Delta E)* \\
p_{j.k}^{exc}(\xi^{'})d(\Delta E)d\xi^{'}\} - \\
\{n_{FC}\int_{U_k}^{E_0^j - U_k}(f_j^{(0)}(t,\vec{r},\xi^{'}))\delta(\xi^{'} - G(\xi,\xi^{'}))\xi\Omega_J^{ChEx}(\xi^{'},\Delta E)* \\
p_j^{ChEx}(\xi^{'})d(\Delta E)d\xi^{'}\} - \{n_{FC}\int_{\Delta E_j^{el}}^{E_0^j - \Delta E_j^{el}}(f_j^{(0)}(t,\vec{r},\xi^{'}))\delta(\xi^{'} - G(\xi,\xi^{'}))\xi\Omega_j^{el}(\xi^{'},\Delta E)* \\
p_j^{el}(\xi^{'})d(\Delta E)d\xi^{'}\}
\end{gathered} \tag{47}
$$

here n_{FC} is a buffer gas concentration (in our case, it is the fissioning component concentration), I is buffer gas ionization potential, $\triangle E_j$ is j-fragment's energy loss, which is equal to the sum of ionization potential and an energy acquired by released electron. Following to [11], we define the delta-function $\delta(\xi_j - G(\xi_j^{'},\xi_j))$ as a function that returns back the value of j-type fragment with velocity ξ_j from another phase volume ($\xi_j^{'} \to \xi_j$), taking into account a definite interaction potential .

Differential cross sections $\Omega_j^{ion}(\triangle E = \varepsilon_{secondary\ electron} + I, \xi_j)$

$$
p_j^{ion}(E_J) = \frac{\Omega_J^{ion}}{\sum\limits_{k=all\ types\ of\ intactions}\Omega_j^k}
$$

and probability of the ionization are defined in analytical form in . Charge transfer in ${}^3He$ plasma accompanied by the following elementary processes:

$$
\begin{gathered}
\{p,T\} +^3 He \to^3 He^+ + H^* \\
{}^3He^+ + 2^3He \to^3 He_2^+ +^3 He^*
\end{gathered} \tag{48}
$$

and equal to:

$$
\begin{gathered}
S_j^{chex}(f_j \to n_{FC}^+) = \int_A^B f_j\Omega_j^{chex}(\xi,\triangle E)\,\mathrm{d}(\triangle E), \\
A = \frac{m_e\left(\xi_e^{He}\right)^2}{2} + U_i^H - U_i^{He}, B = \frac{m_e\left(\xi_e^{He}\right)^2}{2} + U_i^H + U_i^{He}
\end{gathered} \tag{49}
$$

here U_i^H, U_i^{He} the binding energies of hydrogen and helium atoms .

It should be pointed out that the loss of energy in the elastic collisions might be evaluated as follows [10]:

$$\Delta E^{el} = 2\frac{m_j}{M}E_j(1-\cos\chi) \tag{50}$$

here m_j is a mass of fission fragment, M is a colliding atom's mass, $\triangle E$ is an energy loss, E_j is fission fragment's initial energy and $\chi \sim 0.01$.

The recombination term is strongly related to the electron energy distribution function and its time scale relaxation $\frac{pico\, sec}{\varepsilon^2}$ and is equal to:

$$\begin{gathered}\frac{\partial}{\partial t}\left(f_j^{(2)}(t,r,\xi)\right) = -\int_0^{\xi_{emax}} \Omega_{j,e}^{rec}(\xi_j,\xi_e) f_j^{(1)}(t,r,\xi)* \\ *f_e^{(1)}(t,r,\xi)p_e^{rec}(\xi_j-\xi_e)d\xi\end{gathered} \tag{51}$$

The energy distribution function for electrons is presented by equation Equation (**??**) and will be commented in the following way.

$$\begin{gathered}\frac{\varepsilon\partial f_e^{(0)}(t,\vec{r},\vec{\xi})}{\partial t} = S_e^{ff} + \{n_{FC}\int_I^{E_o^j-I} f_j^{(0)}(t,\vec{r},\vec{\xi'})\delta(\xi'-G(\xi,\xi'))\xi\Omega_e^{ion}(\xi',\Delta E)* \\ *p_j^{ion}(\xi')d(\Delta E)d\xi'\} - \{n_{FC}\int_{\Delta E_e}^{E_o^j-\Delta E_e}(f_e^{(0)}(t,\vec{r},\xi'))\delta(\xi'-G(\xi,\xi'))\xi\Omega_e^{el}d(\Delta E)\end{gathered} \tag{52}$$

The first term of this equation presents the electrons released from the outer shell of the atoms undergoing the fissioning process:

$$\begin{gathered}S_e^{ff}(f_e(t,\overrightarrow{r},\overrightarrow{\xi}) = n_{FC}*\Phi*\sigma^{ff}*\delta(\overrightarrow{\xi}-\overrightarrow{\xi}_e^{(0)})\,, \\ \int_o^{\infty}\delta(\overrightarrow{\xi}-\overrightarrow{\xi}_e^{(0)})\,\mathrm{d}\,\overrightarrow{\xi} = 1\end{gathered} \tag{53}$$

The second term of Equation (16) evaluates the rate of primary electrons' formation from all types of fission fragments which were released in fissioning events and equals to:

$$\begin{gathered}S_e^{ion}\left(f_e(t,\overrightarrow{r},\overrightarrow{\xi}\right) = n_{FC}\int_{v_j^{min}}^{v_j}\delta(\xi_e'-G(\xi_e',\xi_e)p_j^{ion}(E_j)d\xi_j'* \\ *\int_I^{E_{e0}-I} f_j(\xi_j)\xi_e'\Omega_e^{ion}(\xi_e',\triangle E_j)d(\triangle E_j)\end{gathered} \tag{54}$$

The recombination term strongly connected with electrons energy distribution function and equals:

$$\begin{gathered}\frac{\partial f_e^{(1)}(t,\overrightarrow{r},\overrightarrow{\xi})}{\partial t} = n_{FC}\int_0^{\xi_{emax}}\Omega_{e,j}^{rec}(\xi_j,\xi_e)f_e^{(1)}(t,\overrightarrow{r},\overrightarrow{\xi})* \\ *f_e^{(0)}(t,r,\xi_e)p_e^{rec}(\xi_j-\xi_e)d\xi_e d(\triangle E)\end{gathered} \tag{55}$$

It should be noted that not only elastic collisions are responsible for heating the buffer gas, regardless of whether or not this gas moves or not, or the gas is in a steady state (motionless). Moreover, the ionization process is accompanied by the recoil process (in crystals, the impact of a colliding particle is redistributed to the whole body, as it is observed in the Mössbauer effect, fluorescence of nuclear resonance). And also not only the ionization process heats the buffer gas due to recoil processes, but it includes all excitation processes. The Boltzmann equation derived in for electrons contains the following restrictions: the electrons energies are such that the cross section of the elastic collision is large compared to inelastic; the frequency of the applied external harmonic electric field, is much higher than the own plasma in the discharge region.

Attempts to replace an unknown ionization rate by a recombination rate may also lead to the uncontrolled discrepancies [22], [23]. The recombination process directly depends on the electrons' energy distribution, which in its own turn, depends on the evolution of primary electrons' distribution created precisely by fission fragments in fissioning plasma. Taking into the consideration quick,slow time scales the time dependent energy distribution functions for fission fragments and electrons might be obtained from equations (44),(52):

$$\begin{gathered} f_e(t,\xi) = n_0^e \left(\frac{m_e}{I}\right)\delta(\xi-\xi_0) + f^{pe}(\xi) - f^{pe}(\xi)e^{-\frac{t}{\tau}} + \\ + f^M(\xi) - f^M(\xi)e^{-\frac{t}{\varepsilon\tau_1}} f^M(\xi) = n_e\left(\frac{m_e}{kT_e}\right)^{\frac{3}{2}} e^{-\frac{m_e\xi^2}{kT_e}} \end{gathered} \tag{56}$$

here coefficients τ and τ_1 are respectively equal:

$$\begin{gathered} \frac{1}{\tau} = n_{FC}\int_I^{\xi_{max}} \Omega^{ion}(\triangle E,\xi)\xi d(\triangle E) + \\ + \sum_k \Omega_k^{exc}((\frac{m_e\xi^2}{2} + I_k), \xi + \sqrt{\frac{I_k}{m_e}})(\xi + \sqrt{\frac{I_k}{m_e}}) \end{gathered} \tag{57}$$

And the second,so called "slow" time scale:

$$\begin{gathered} \frac{1}{\tau_1} = \int_I^{E^{max}} \Omega_e^{ion}(\Delta E,\xi)\xi d(\Delta E) + \\ + \sum_k \Omega_e^{exc}((\frac{m_e\xi^2}{2} + I_k), \xi + \sqrt{\frac{I_k}{2m_e}})(\xi + \sqrt{\frac{I_k}{2m_e}}) + \\ + \int_{E^{min}}^{E^{max}} \Omega_e^{el}(\Delta E,\xi)\xi d(\Delta E) + \int_0^{E^{max}} \Omega_e^{rec}(\Delta E,\xi)\xi d(\Delta E) \end{gathered} \tag{58}$$

Following to [10] and [2] the primary electrons energy distribution equals:

$$\begin{gathered} f_e^{pe}(\varepsilon) = n_e^0 \left\{\frac{m_e}{E_q}\right\}^{\frac{3}{2}} * G(\varepsilon) \\ G(\varepsilon) = \frac{I^3}{(I+\varepsilon)^2 E_q} \left\{ \frac{\frac{I+\varepsilon}{I} + \frac{4}{3}(1 - \frac{I+\varepsilon}{I} Ln(2.7 + (\frac{E_q - I - \varepsilon}{I})^{0.5})}{1 + \frac{1}{3} Ln(2.7 + (\frac{E_q - I}{I})^{0.5}} \right\} \\ E_q = \frac{m_f f(\xi_{ff}^0)^2}{2} \end{gathered} \tag{59}$$

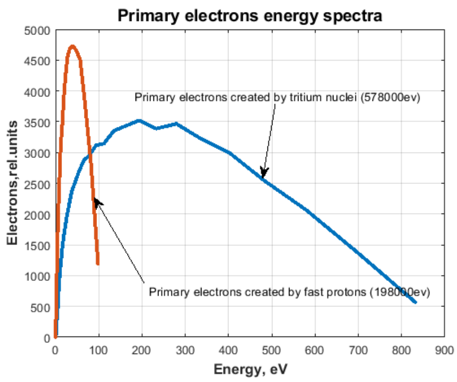

Figure 1 – Primary electrons energy spectra,calculated by Monte Carlo technique, [24]

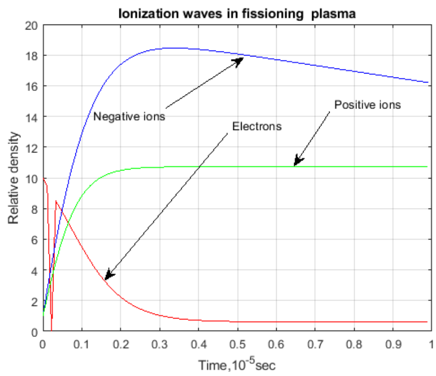

Figure 2 – Ionization waves in $^{235}UF_6$ fissioning plasma

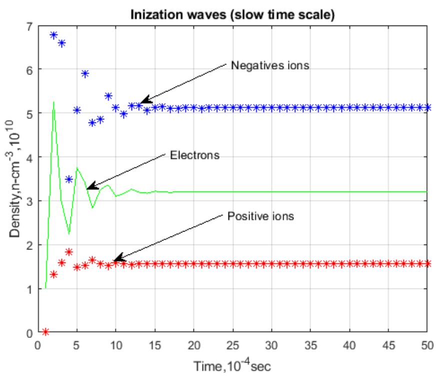

Figure 3 – Ionization waves in $^{235}UF_6$ fissioning plasma

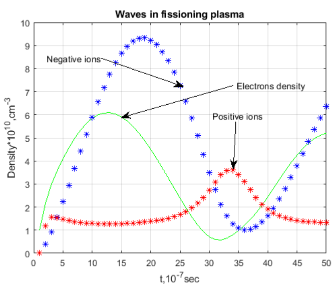

Figure 4 – Ionization waves in $^{235}UF_6$ fissioning plasma

Conclusions.

- Self-consistent Boltzmann equations for fission fragments and electrons are derived.Electrons energy spectra are directly connected with primary electrons energy distribution of fission fragment energy spectra
- Time dependent solution of Boltzmann equations is presented
- Hydrodynamic equations for electrons in fissioning plasma are derived
- Ionization waves in fissioning plasma are presented in different time scales:fast and slow scales

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Бөлінетін плазмадағы гидродинамикалық теңдеулер және иондалған толқында

Аннотация. Бөліну фрагменттері үшін өзара үйлесетін Больцман теңдеулері және құрылған бастапқы электрондар әлсіз иондалған тығыз үшін анықталады бөліну фрагменттерімен сәулеленген плазма. Осы теңдеулер негізінде плазмадағы жылдам бөлшектердің энергия түзілуінің кинетикасы зерттелген. Бөліну фрагменттеріне арналған тұрақты аналитикалық шешімдер және біріншілік электрондардың бөлінетін газ тәріздес материалдар үшін энергия тарату функциялары нейтрондар ағынымен сәулеленген. Нәтижелер Монте-Карло әдісі бойынша энергетикалықспектрлерді есептеумен салыстырылатын болады.

Түйін сөздер: Больцман теңдеуі, бөліну фрагменті, бөлінетін плазма, иондану толқыны, гидродинамикалық теңдеулер.

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Уравнения гидродинамики и ионизационные волны в делящейся плазме

Аннотация. Связанные самосогласованные уравнения Больцмана для осколков деления и рожденных первичных электронов определены для слабоионизованной плотной плазмы, облученной осколками деления. На основе этих уравнений исследована кинетика образования энергии быстрых частиц в плазме. Найдены и проанализированы стационарные аналитические решения для осколков деления и функций распределения энергии первичных электронов для плазмы гелия-3, облучаемой потоком нейтронов. Результаты сравниваются с расчетами энергетических спектров методом Монте-Карло.

Ключевые слова: уравнение Больцмана, осколок деления, делящаяся плазма, волна ионизации, уравнения гидродинамики.

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