Л.Н. Гумилев атындағы Еуразия ұлттық университетінің ХАБАРШЫСЫ.
ISSN: 2616-6836. eISSN: 2663-1296

ФИЗИКА. АСТРОНОМИЯ СЕРИЯСЫ/ PHYSICS. ASTRONOMY SERIES / СЕРИЯ ФИЗИКА. АСТРОНОМИЯ

IRSTI 29.05.29
Scientific article

https://doi.org/10.32523/2616-6836-2024-147-2-20-27

Investigation of vertex a_1→VP for hadronic τ decays

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Abstract. In the framework of U(3) symmetric quark model, triangular diagrams are calculated that describe the strongly interacting vertices $a_1 \to \rho\pi$ and $a_1 \to K$^* K. The divergences arising when considering quark loops are regularized by covariant cut off parameter. Based on four-particle hadronic τ decays, the quark structure of the a_1 resonance with quantum numbers J^{PC}=1^(++) was studied. Theoretical estimates of intermediate channels with the a_1 meson for the partial decay widths of $\tau \to \pi\pi\pi\nu_\tau$, $\tau \to KK\pi\nu_\tau$ and $\tau \to KK\eta\nu_\tau$ are obtained. Contributions from contact channels describing the direct production of 3π, $KK\pi$ and $KK\eta$ mesons by τ lepton current are taken into account. The interference of contact and intermediate axial-vector channels in determining the integral decay widths is studied. The matrix elements of the processes are obtained in the leading approximation of $1/N_C$ expansion.It is shown that calculations on τlepton mesonic decays confirm the quark-antiquark structure of the a_1 meson and clarify the role of the intermediate non-strange axial-vector channel in determining the integral partial decay widths. The obtained results are compared with experimental data by BaBar and Belle collaborations at the BEPC II and KEK lepton colliders.

Keywords: hadron structure, quark fields, meson interactions.

Received 02.05.2024. Revised 19.05.2024. Accepted 27.05.2024. Available online 30.06.2024

Introduction

Description of the mass spectrum and interactions of the axial-vector meson field a_1 is an important task in the non-strange *SU(2)×SU(2)* sector of low-energy QCD phenomenology. The properties and decays of the a_1 meson field were studied on the basis of inelastic reactions $\pi^- p \to \pi^- \pi^{\wedge -} \pi^{\wedge +} p$ with a beam energy of 190 GeV/c by the COMPASS collaboration [1]. In the CLEO and LCHb experiments at the Large Hadron Collider, branching fractions of heavy $D^0 \to K^{\mp} \pi^{\pm} \pi^{\pm} \pi^{\mp}$ meson decays measured, where the analysis of obtained data indicates the decisive role of intermediate channels with the a_1 meson [2, 3]. In addition, the parameters of the a_1 meson can be studied in the three-pion final state of the τ lepton decay. The latter comes without hadron impurity and, therefore, provides good accuracy. Relevant studies were carried out in [4,5].

The a_1 meson is characterized by quantum numbers $J^{PC}=1^{++}$. It isgenerally accepted that this meson is a bound state of $\bar{q}q$ quark fields. Quark fields are associated with the operator $\gamma_\mu \gamma_5$. The mass spectrum of the a_1 meson in the ground and first excited states is described in the quark model [6].

In the present paper, we give a theoretical description of decays $a_1 \to \rho\pi$ and $a_1 \to K^* K$. The matrix elements of the indicated decays are calculated in the one-loop quark approximation in the leading order of expansion N_C, where N_C is the number of colors in QCD [6]. The estimates were obtained for the partial widths of $\tau \to \pi\pi\pi\nu_\tau$, $\tau \to KK\pi\nu_\tau$ and $\tau \to KK\eta\nu_\tau$ with different charge combinations which proceed predominantly through intermediate channels containing decays $a_1 \to \rho\pi$ and $a_1 \to K^* K$.

Methods

To describe the strong interaction vertices of theme son a_1 with the ρπ and K * K meson fieldsuse the effective Lagrangian obtained on the basis of a model with four-quark interaction [6,7]. The decay $a_1 \to \rho\pi$ is described by the Feynman diagram shown in Fig. 1. The meson fields in the vertex a_1 ρπ are coupled by the u and d quark fields. In the case of a_1 K* K, the stranges quark field is in the internal lines. The Lagrangian determined the interaction of meson fields with quarks is represented in the following form

$$\Delta L_{int} = \bar{q}[1/2\, \gamma^\mu \gamma^5 \lambda^{a1}_{\pm,0} g_\rho a^{\pm,0}_{1\mu} + i\, \gamma^5 \lambda^{\pi}_{\pm,0} g_\pi \pi^{\pm,0} (1)$$
$$+1/2\ \gamma^\mu \lambda^{\rho}_{\pm,0} g_\rho \rho^{\pm,0}_\mu + 1/2 \gamma^\mu \lambda^K g_{K*} K *_\mu + i\, \gamma^5 \lambda^K_{\pm,0} g_K K^{\pm,0}]q,$$

where a_1, π, ρ, K * and K are the meson fields; γ_5, γ_μa re Dirac matrices in four-dimensional Minkowski space; *q* an r $\bar{q}$ ar *eu, d* and *s* quarktriplets with constituent masses $m_u \approx m_d$= 270MeV and m_s= 420MeV [7]; λare liner combinations of Gell-Mann matrice

$$\lambda^K_- = \frac{\lambda_4 - i\,\lambda_5}{\sqrt{2}},\ \ \lambda^K_0 = \frac{\lambda_6 + i\,\lambda_7}{\sqrt{2}},$$
$$\lambda^\rho_- = \frac{\lambda_1 - i\,\lambda_2}{\sqrt{2}},\ \ \lambda^\rho_0 = \frac{\sqrt{2}\lambda_0 - \lambda_8}{\sqrt{2}}.$$

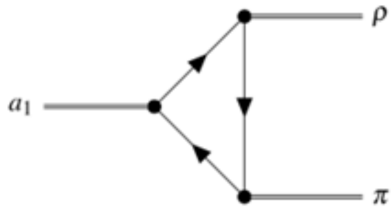

Figure 1. Triangle quark diagram describing decay $a_1 \to \rho\pi$

The coupling constants g_ρ, g_K, g_π and g_{K^*} determine the interactions of mesons with quarks [7]. These constants determine the renormalization of meson fields and are expressed through logarithmically divergent integrals

$$g_{K*} = \sqrt{3/2\, I_{11}},\ g_K = \sqrt{Z_K/4\, I_{11}}, (2)$$

$$g_\rho = \sqrt{3/2\, I_{20}},\ g_\pi = \sqrt{Z_\pi/4\, I_{20}},$$

where

$$I_{n_1 n_2} = -i\frac{N_C}{(2\pi)^4}\int \frac{\Theta(\Lambda_4^2 + k^2)}{(m_u^2 - k^2)^{n_1}(m_s^2 - k^2)^{n_2}} d^4k. \quad (3)$$

Using the Lagrangian (1) above, one can construct a vertex with a triangular loop. Consideration of this vertex leads to an integral of the following form

$$I_{21}^{a_1\rho\pi} = N_C \int \frac{Tr\left[\gamma^\mu\gamma^5\left(\hat{k} + \widehat{p_\rho} + m_s\right)\gamma^\nu\left(\hat{k} + m_u\right)\gamma^5\left(\hat{k} - \widehat{p_\pi} + m_u\right)\right]}{\left[\left(k + p_\rho\right)^2 - m_u^2\right][k^2 - m_u^2][(k + p_\pi)^2 - m_u^2]} \frac{d^4k}{(2\pi)^4}. \quad (4)$$

When calculating this integral, we use one-loop approximation [6]. As a result, obtain the following decay matrix element of $a_1 \to \rho\pi$

$$M(a_1\rho\pi) = -i\, F_\pi g_\rho^2 Z_\pi \varepsilon_\mu(a_1) g_{\mu\nu}\varepsilon_\nu(\rho), \quad (5)$$

where ε_μ (a_1) and ε_ν (ρ) are polarization vectors.

In a similar way, we obtain the amplitude describing the decay aa_1 1→K^* K with the participation of strange mesons

$$M(a_1 K^* K) = -i\, \frac{3 g_\rho g_K}{\sqrt{2} g_{K*}} (3m_u - m_s)\, \varepsilon_\mu(a_1) g_{\mu\nu}\varepsilon_\nu(K*). \quad (6)$$

Results and discussions
The contributions of axial-vector channels to hadronic τ decays

In this chapter we consider mesonic τ decays $\tau \to \pi\pi\pi\nu_\tau$, $\tau \to KK\pi\nu_\tau$ *and* $\tau \to KK\eta\nu_\tau$, where the vertices $a_1 \to \rho\pi$ and $a_1 \to$K^* K obtained above contribute as intermediate channels. The diagrams corresponding to these channels are shown in Fig. 2 and 3. These channels are considered in conjunction with the contact diagram with the axial-vector part of the W boson, when the final products are produced directly by the lepton current. The corresponding diagram for the contact channel is shown in Fig. 4. The decay of $\tau \to W\nu_\tau$ is described by the electroweak Lagrangian of the standard model (Weinberg – Salam theory) [8,9]. The parts of diagrams describing W boson interaction with meson fields are obtained within the effective quark model.

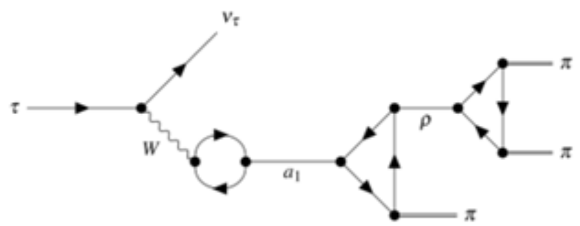

Figure 2. Feynman diagram describing channel with a_1 meson for the decay $\tau \to 3\pi\nu_\tau$

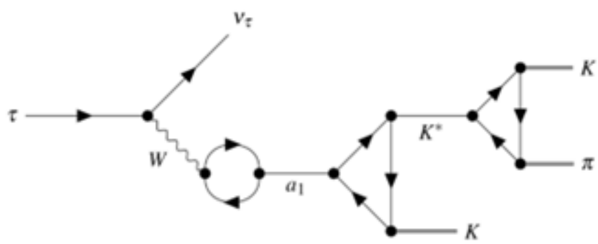

Figure 3. Feynman diagram describing channel with a_1 meson for the decay $\tau \to KK\pi\nu_\tau$

The decay amplitude $\tau \to \pi\pi\pi\nu_\tau$ can be represented as the product of the lepton current with the hadronic current in the following form:

$$M\,(\tau \to \pi^+\pi^-\pi^-\nu_\tau) = -iF_\pi G_F V_{ud} L_\mu M_{A\mu}, \quad (7)$$

Where $L_\mu = \bar{\nu}_\tau \gamma_\mu (1-\gamma_5)\tau$ is the lepton current, G_F is the Fermi constant, V_{ud} is the element of the Cabibbo-Kobayashi-Maskawamatrix.

The total contribution from contact term and intermediate axial-vector channel reads

$$M_{A\mu} = Z_\pi g_\rho^2 \big(g_{\mu\nu} + ((q^2 - 6m_u^2)g_{\mu\nu} - q_\mu q_\nu)BW_{a_1}\big)BW_\rho (p_{\pi^+} - p_{\pi^-}^{(1)})_\nu + (p_{\pi^-}^{(1)} \leftrightarrow p_{\pi^-}^{(2)}), \quad (8)$$

where $p_\pi^{\ +}$, $p_\pi^{\ (1)}-$ and $p_\pi^{\ (2)}$ are the momentum of charged pions in the final state.

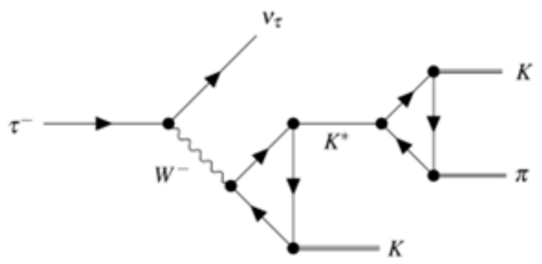

Figure 4. The contact diagram describing decay $\tau \to KK\pi\nu_\tau$

The transition of the W boson into intermediate a_1 meson has a gauge-invariant structure

$$g^{\mu\nu}\,[q^2 - 6m_u^2] - q_\mu q_\nu \quad (9)$$

The a_1 meson in the intermediate state is described by the Breit-Wigner propagator

$$BW_M = \frac{g_{\mu\nu}}{M_{a_1}^2 - q^2 - i\sqrt{q^2}\Gamma_{a_1}}. \quad (10)$$

Table 1. Calculated branching fractions (Br) for hadronic τ decays. Comparison of theoretical results with experimental data of BaBar and Belle collaborations is presented.

Br(×10⁻³)					
Mode	$\tau \to \pi^- \pi^+ \pi^- \nu_\tau$	$\tau \to \pi^0 \pi^0 \pi^- \nu_\tau$	$\tau \to K^- K^0 \pi^0 \nu_\tau$	$\tau \to K^- K^+ \pi^- \nu_\tau$	$\tau \to K^- K^0 \eta\nu_\tau$
Theor	90.5	91.5	1.46	0.4	0.0014
PDG [10]	90.2±0.5	92.6±1.0	1.50±0.07	1.43±0.02	<0.009
BaBar	88.3±1.3 [11]	–	–	1.34±0.03 [11]	–
Belle	84.2±2.5 [4]	-	1.49±0.07 [12]	1.55±0.01 [4]	–

To calculate the decays $\tau \to KK\pi\nu_\tau$ *and* $\tau \to KK\eta\nu_\tau$ consider the vertices of W boson interaction with meson fields presented Fig.3. When considering these vertices, use approximation in one-loop. The full matrix elements for axial-vector channels with an intermediate a_1 meson can be found in paper [13].

The obtained results for the decay channels $\tau \to 3\pi\nu_\tau$, $\tau \to KK\pi\nu_\tau$ and $\tau \to KK\eta\nu_\tau$ are given in Table 1. The uncertainty of theoretical calculations is estimated at 15% [6,7].

Conclusion

In the present paper, the vertices $a_1 \rightarrow \rho\pi$ and $a_1 \rightarrow K^* K$ going through the triangular quark loop are calculated (Fig.1). The amplitudes of the processes considered are obtained using chiral Lagrangians in leading order of $1/N_C$, where N_C is the number of colors in QCD. Estimates are obtained for the branching fractions of $\tau \rightarrow 3\pi\nu_\tau$, $\tau \rightarrow KK\pi\nu_\tau$ and $\tau \rightarrow KK\eta\nu_\tau$ taking into account the contact channel and channels with the intermediate meson a_1. Interference between contact and axial vector channels is always negative due to the negative value of the Breit-Wigner propagator for most of the energy range $(\sum M_P)_2 \leq q^2 \leq M\tau^2$.

Our calculations for the considered hadronic τ decays confirm the quark-antiquark structure of the a_1 meson and clarify the role of intermediate channels with the nonstrange a_1 meson in determining the partial widths of the decays. Model predictions for the branching fraction do not contradict the experimental data of BaBar [11] and Belle [4,12].

Acknowledgement

The author thanks collaborators prof. M.K. Volkov and A.A. Pivovarov. This work was supported by the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan Grant no. BR20280986.

The contribution of the authors

Kanat Nurlan – writing the text and critically revising its content, approval of the final version of the article for publication, agreement to be responsible for all aspects of the work, proper examination and resolution of issues related to the reliability of the data.

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τ лептонның адронды ыдыраулары үшін $a_1 \to VP$ шыңын зерттеу

Аңдатпа. U(3) симметриялы кваркті модел шеңберінде пәрменді әсерлесу жүзеге асатын $a_1 \to \rho\pi$ және $a_1 \to K^* K$ шыңдарын сипаттайтын үшбұрышты диаграммалар есептелді. Кварткы диаграммаларды қарастырған кезде пайда болатын жинақсыздық ковариантты кесу параметрі арқылы реттелген. τ – лептонның төрт бөлшекті адрондық ыдырауы негізінде $J^{PC}=1^{++}$ кванттық сандары бар a_1 резонансының кварк құрылымы зерттелді. a_1 мезоны бар аралық арналардың теориялық бағалары $\tau \to \pi\pi\pi\nu_\tau$, $\tau \to KK\pi\nu_\tau$ және $\tau \to KK\eta\nu_\tau$ жартылай ыдырау ендері үшін есептелді. τ – лептонды токтан 3π, KKπ және KKη мезондарының тікелей тууларын сипаттайтын байланыс арналарының үлестері есепке алынды. Ыдыраулардың интегралдық ендерін анықтауда контактілі және аралық аксиал-векторлы арналардың интерференциялары зерттелді. Процестердің матрицалық элементтері $1/N_c$ кеңеюінің жетекші жуықтауында алынды. τ – лептонның мезондық ыдыраулары бойынша есептеулер a_1 мезонының кварк-антикварк құрылымын растайтыны және ыдыраулардың ішінара интегралдық ендерін анықтаудағы акссиал-векторлық аралық арнаның рөлін нақтылайтыны көрсетілген. Алынған нәтижелер BEPC II және KEK лептон коллайдерлерінде алынған BaBar және Belle тәжірибелік деректерімен салыстырылды.

Түйін сөздер: адрондардың құрылымы, кваркты өрістер, мезондардың әсерлесулері.

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Исследование вершины $a_1 \to VP$ для адронных τ распадов

Аннотация. В рамках U(3) симметричной кварковой модели вычислены треугольные диаграммы, описывающие сильно взаимодействующие вершины $a_1 \to \rho\pi$ и $a_1 \to K^* K$. Расходимости, возникающие при рассмотрении кварковых петель регуляризуются ковариантным обрезанием. На основе четырех частичных адронных распадов тау-лептона исследована кварковая структура резонанса a_1 с квантовыми числами $J^{PC}=1^{++}$. Получены теоретические оценки промежуточных каналов с мезоном a_1 для парциальных ширин распадов $\tau \to \pi\pi\pi\nu_\tau$, $\tau \to KK\pi\nu_\tau$ и $\tau \to KK\eta\nu_\tau$. Учтены вклады от контактных каналов, описывающие прямое рождение мезонов 3π, $KK\pi$ и $KK\eta$ тау-лептонным током. Исследованы интерференций контактных и промежуточных аксиально-векторных каналов в определении интегральных ширин распадов. Матричные элементы процессов получены в лидирующем приближений разложения $1/N_C$. Показано, что расчеты по мезонным распадам тау-лептона подтверждают кварк-антикварковую структуру мезона a_1 и проясняет роль промежуточного нестранного аксиально-векторного канала в определении парциальных интегральных ширин распадов. Полученные результаты сравниваются с экспериментальными данными BaBarи Belle, полученные на лептонных коллайдерах BEPCII и KEK.
Ключевые слова: структуры адронов, кварковые поля, взаимодействия мезонов.

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